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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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MAY 1926

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIII MAY 1926 NUMBER 4

CRITICAL POTENTIALS AND SPECTRA OF OXYGEN

By L. L. LOCKROW

ABSTRACT

The spectra.—The *O*I and *O*II line spectra and the band spectra of oxygen in the visible and ultra-violet regions as excited in a low-voltage arc discharge-tube were obtained and the critical potentials for excitation were noted.

Critical potentials.—Ionization of the molecule was found to occur at 16.1 volts. The resonance potential at 9 volts formerly assigned to the atom is also a radiating potential for the molecule. Dissociation of the molecule and simultaneous ionization of one atom occurs at 19.5 volts. From this value and that for ionization of the atom, 13.5, the value of 6 volts is obtained for the dissociation potential of the oxygen molecule. This corresponds to an energy of dissociation of 138,000 calories per gram molecule. Doubly ionized atoms first appear in the discharge at about 50 volts.

Energy-levels for band spectra.—Energy-levels for the electronic states were observed at 16.1, 19.2, and 21 volts. To these we may add the level at 9 volts, as that for the neutral molecule.

This paper deals with experiments with oxygen, from which information has been obtained concerning molecular and atomic critical potentials and their interpretation, as based on the attendant spectra, and is a companion to papers by Duffendack[1] and by Duncan[2] in which are described researches on hydrogen and nitrogen.

A number of observers have reported values for the radiating, resonance, and ionization potentials of oxygen. Franck and Hertz, Hughes and Dixon, and Bishop found a value of about 9 volts for a potential which has recently been ascribed to radiation.[3] Mohler

[1] *Physical Review*, **20**, 100, 1922; *Astrophysical Journal*, **60**, 122, 1924.

[2] *Ibid.*, **62**, 145, 1925.

[3] Hughes, *Bulletin of the National Research Council*, No. 10, 1921.

and Foote[1] found the values 7.9 and 15.5 volts for resonance and ionization potentials, respectively, and Boucher[2] found 8.0 and 14.0 volts for the same quantities. In a study of the spectral relations in oxygen, Hopfield[3] found lines leading to the values 9.11 and 13.56 volts for the potentials of the atom. Recently Mackay[4] found the value of 12.5 volts for an ionization potential, using a tube in which the electrons were furnished by a photo-electric source. He also found ionization setting in at 16 volts. From experiments made with a positive-ray discharge-tube, H. D. Smyth[5] concluded that in oxygen molecular ions were produced at about 15.5 volts and doubly and singly charged atomic ions at about 22.5 and 23 volts, respectively.

Apparatus.—The discharge-tube (Fig. 1) used during the spectroscopic investigation was provided with a quartz plate cemented to a side tube. In the other experiments no side tube was used. The cathode consisted of a spiral of 0.25 mm platinum wire wound on an arbor 0.51 mm in diameter. The spiral was heated in concentrated nitric acid, ammonium hydroxide, and distilled water in turn to clean it, and then it was coated with a mixture of barium, calcium, and strontium oxides. The anode was made of a nickel strip bent into the form of a U of such a size as to make the distance from filament to anode about 3 mm. A liquid-air trap was placed near the tube to prevent the access of mercury vapor, and the tube was thoroughly baked out during the exhausting process. The apparatus was evacuated by a mercury-vapor condensation pump. Oxygen was first prepared by the electrolysis of a solution of barium hydroxide and later by heating potassium permanganate in a side tube sealed on the pump system. The gases from the two sources gave identical spectra.

The circuit used was essentially that shown in Figure 1. The anode voltage as read on the voltmeter is that measured from the negative end of the filament. There is a correction to be applied

[1] *Journal of the Optical Society of America*, **4**, 49, 1920.

[2] *Physical Review*, **19**, 189, 1922.

[3] *Astrophysical Journal*, **59**, 114, 1924.

[4] *Philosophical Magazine*, **46**, 828, 1923; *Physical Review*, **24**, 319, 1924.

[5] *Proceedings of the Royal Society*, A, **105**, 116, 1924.

because of the voltage-drop along the filament leads and along the filament itself. The middle of the filament is most effective in emission so the correction is made by subtracting one-half of the voltage-drop along the filament and its leads from the reading of the anode

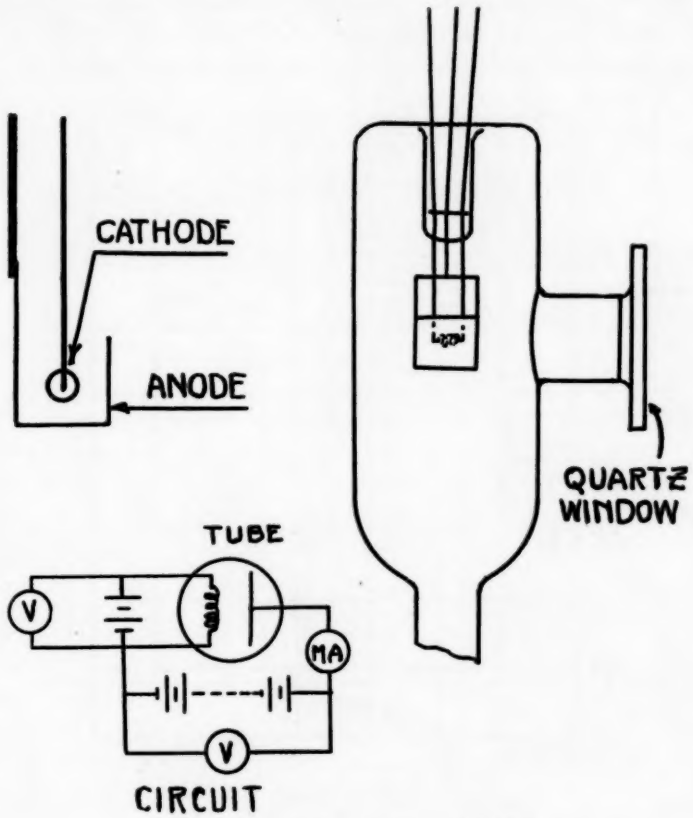


FIG. 1.—Tube, electrodes, and circuit

voltmeter. In discussing the development of spectra this corrected voltage is the one given.

Determination of arcing potential.—Oxide-coated platinum, bare tungsten, and molybdenum filaments were used, most of the work being done with the oxide-coated filaments. Two series of tests were made on the low-voltage arc with a tube of the type described. The conditions of gas pressure and filament temperature influenced the

potential at which the curves broke; therefore one series of tests was made with constant filament temperature and different gas pressures to determine the pressure at which an arc could be maintained at the lowest voltage, and the other was made at the optimum pressure but with varying filament temperature.

In Figure 2 is shown a current-voltage curve for the arc typical of those from which the values for the arcing potentials were ob-

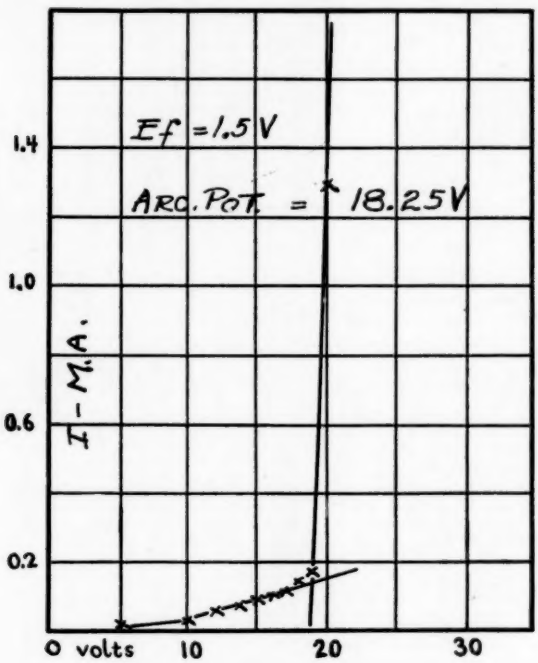


FIG. 2.—Current-voltage curve

tained. All curves showed a distinct break, and the values obtained with decreasing voltage agreed within the limit of error with those obtained with increasing voltage.

The variation in arcing potential with gas pressure is shown in Figure 3*a*. At low pressures the arcing potential is high. It decreases to a minimum at 0.7 or 0.8 mm, and then it increases with further increase in pressure. At low pressures the mean free path between effective collisions is large compared to the spacing of the

electrodes, and there are not many collisions of electrons with molecules. At the optimum pressure the electrons make the largest number of effective collisions during their travel from cathode to anode. At higher pressures the energy of the electrons is dissipated

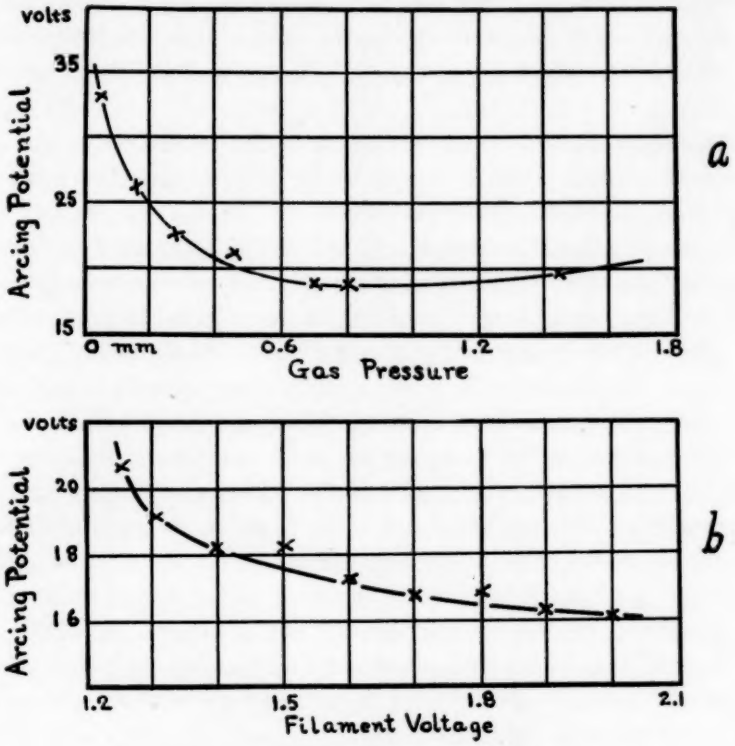


FIG. 3.—Variation in arcing potential:
a, with gas pressure;
b, with filament voltage at optimum pressure.

by inelastic collisions and momentum exchanges with the molecules.

The variation of the arcing potential with the filament temperature at the optimum pressure is shown in Figure 3*b*. The lowest value obtained for the arcing potential was 16.1 volts, corrected for filament drop. The curve shows that a slightly lower value may be obtained for the arcing potential, but that it cannot be far different from that given above. Below this minimum arcing potential of

16.1 volts not enough positive ions are formed to show on the current-voltage curves and accordingly this value is taken to represent the critical potential for an ionization process.

Experiments with filaments of tungsten and molybdenum gave essentially the same results. With tungsten the lowest break came with a corrected voltage of 15.5 volts. The molybdenum filaments had only a very short life but their behavior was similar to that of tungsten.

Spectroscopic studies.—In this part of the investigation the determination of the critical potentials for the excitation of line and band spectra is the important feature.

In determining the value of any one critical potential a number of spectrograms were obtained at small steps of voltage under conditions which gave the largest possible electron current in the tube and the greatest sensitivity of the spectrograph. Since the intensity of any particular radiation increases rapidly after its critical potential has been exceeded, it is possible to obtain a definite value for this quantity. Whenever it was possible, several photographs were examined so as to narrow the limits of the voltages for the appearance and non-appearance of the radiation. Due to the lack of homogeneity in the electron stream because of the initial velocities of the electrons and because of the voltage-drop along the filament, it is not possible to excite one radiation without another if the difference in the critical voltages is small. In one case, however, photographs taken with but 0.3 volt steps did show the progressive extension of a band system, as would be expected.

The "OI" line spectrum.—The low-voltage arc type of excitation favors the band spectra. Relatively few of the lines appear, and these are in the region 6500 to 3600 A. There may be other lines present which are obscured by the bands. The line 4368 A, $1S—3P$, of the singlet principal series is the strongest line found in the spectral region studied. The next line of this series, 3692 A, $1S—4P$, is considerably weaker. No other lines of the singlet series appear with any certainty. The triplet 3947 A, $1s—3p_{123}$, of the principal series is second in order of intensity. Most of the lines of the triplet sharp and diffuse series have been identified. The lines have intensities corresponding to those given by others, with the exception

of the triplet at 4589 as observed on a 100-volt plate. This line is the last one observed in the series and has an intensity of 10 as compared with an intensity of 2 for the previous line of the same series. In this case it seems probable that there is a line of the enhanced spectrum superimposed on the sharp triplet.

The "OII" line spectrum.—Photographs taken at 45 and 50 volts showed certain lines appearing at the latter voltage which were not present before. These lines have been identified as belonging to the *O*II or first enhanced spectrum. At 100 volts most of the lines given by Eder and Valenta[1] as being of the spark or *O*II spectrum of oxygen between 4705 and 4180A were observed with about the correct intensity. Two lines not given by Eder and Valenta appeared at 4648 and 4227 with intensities of 10 and 8. This latter line may be confused with the strong calcium line at a slightly shorter wavelength. In this case it is probably due to the excitation of calcium from the filament.

Development of the line spectra.—The first line, 4368A, 1S—3P, of the singlet series appears at 16.7 volts, faintly. Then appear the lines at 3947, 6156, 5330, 4968, and 5436, all triplets, in this order. This development is one of intensity, however, and not of energy-levels, for the intensities of the lines are graded, and with longer exposures the lines will appear at lower voltages.

The intensities of both 4368 and 3947A increase markedly between 18.8 and 19.8 volts on one plate and between 19.2 and 20.2 volts on another. The explanation of this is as follows: We obtain the atomic lines down to the critical ionization potential 16.1 volts by cumulative action which only occurs with a relatively heavy anode current. In this cumulative process the first collision of an electron with a molecule may dissociate it so that a second electron colliding with one of the atoms is able to ionize it, or atoms may be produced by dissociation in impacts of the second kind with excited molecules and these resulting atoms ionized by a subsequent electron impact. At a higher voltage the electron is able to dissociate the molecule and ionize one of the atoms in a single collision, or ionize the molecule and excite it so that it will dissociate upon collision with a neutral molecule. When this voltage is reached the

[1] *Atlas typischer Spectren*, 85.

atomic ionization will be far more intense because the probability of this kind of process is far greater, and consequently the atomic lines increase in intensity. From the figures given above, we conclude that the value 19.5 volts best represents the voltage at which occur the dissociation and ionization processes as the result of a single electron encounter.

The low intensity of the atomic lines below 19.5 volts gives further credit to the conclusion that the arc is not maintained by atomic ions produced by cumulative action, but rather by molecular ions produced directly. Since the probability of cumulative action is normally slight, the total number of atomic ions produced when there is no arc is very small because the portion of the path of an electron in which effective collisions can occur is short. When there is an arc, most of the drop in potential between anode and cathode occurs near the cathode, the path for an effective collision becomes much larger, and more atomic ions are produced by cumulative action. The first enhanced lines appear at 50 volts, and so we may take this as an approximate figure for the critical potential for the double ionization of the oxygen atom.

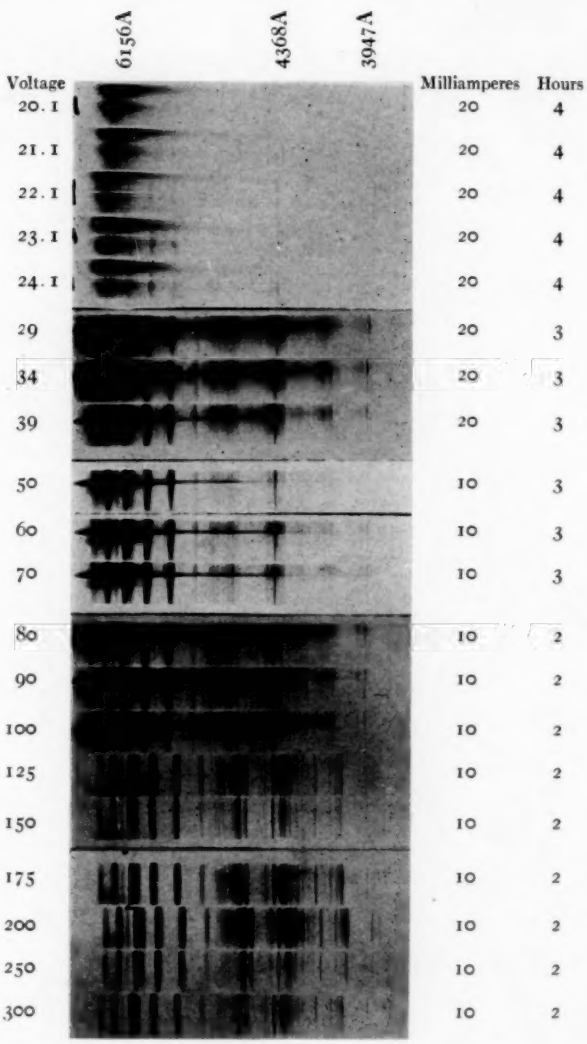
The development of the *O*I, *O*II, and band spectra is shown on Plate XIII for the visible region from 20.1 to 300 volts. The currents and times of exposure of the plates are given. The enhanced lines are first visible at 60 volts. At 300 volts, they are more intense than the *O*I lines and relatively far more intense than the bands.

Excitation at 1000 volts.—A tube was made with an oxide-coated filament of the usual type but with the cathode placed at a distance of 3 cm from the anode. A bulb of about 6-cm diameter was blown in the tube for clearance around the elements.

In one case a discharge was started and maintained in the tube at 700 volts and 5 ma,[1] and gas pressure of about 0.1 mm, without heating the filament by the battery. While the discharge was passing the filament fluoresced, giving off a yellow light. In another case the discharge was self-maintained with 1000 volts, a current of 3.5 ma, and a gas pressure of 0.36 mm. A discharge similar but for a shift in the position of the glow was obtained with the filament

[1] Abbreviation for milliampere.

PLATE XIII



DEVELOPMENT OF OXYGEN SPECTRA IN THE VISIBLE REGION

heated. When viewed from a point along the direction of the filament with the filament heated, the glow seemed to consist of two distinct parts separated by a sharp line (see Fig. 4). The glow around the cathode was of a reddish-violet color while the remainder was the yellow-green characteristic of the low-voltage discharge. Ex-

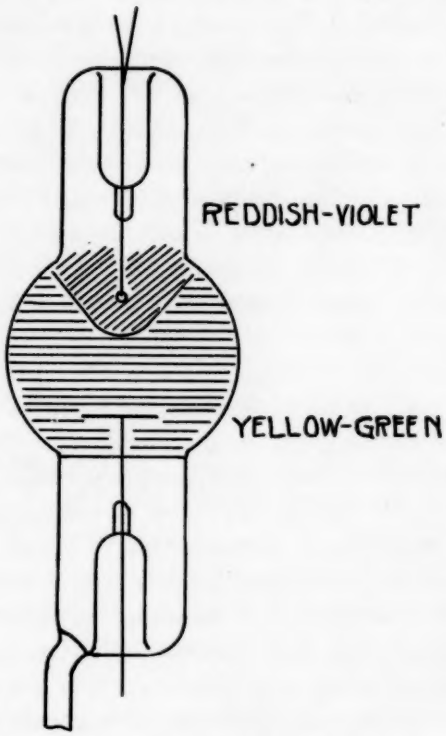


FIG. 4.—Glow at 1000 volts

amination with a spectroscope revealed that most of the light of the cathode glow came from the triplet at 6156 and the *O*II lines. The green glow came from the negative bands in the red and green.

When the glow was self-sustaining, the space of red glow was larger and less clearly defined than when the filament was heated and the green glow occupied only a small space around the anode. As the gas was "cleaned up," the red glow extended and the green glow faded out. It was necessary to admit gas every 15 or 20 minutes under these conditions.

Photographs showed the *O*I and *O*II spectra to be present in both glows. Only a trace of the negative bands was found in the red glow, but in the green glow they were strong. We may conclude from this that in the cathode glow the concentration of molecules is much smaller than in the remainder of the tube.

Energy-levels in the band spectra of oxygen.—The emission of band spectra by molecules is brought about by transitions in state of their electronic, vibrational, and rotational energy from greater to less energy.[1] If details of the spectra are well-enough known it should be possible to construct a complete diagram of the energy-level for the molecular spectra similar to those for atomic spectra.[2] In no case has the complete diagram been worked out, but for several elements, as oxygen, some of the levels have been evaluated. For oxygen three levels for the electronic state have been determined. The levels of the vibrational state are too close to be found by a study of the critical excitation potentials.

The best known of the emission bands of oxygen are those which comprise what will be called the "second negative band system,"[3] extending from red to green at 6853–6567, 6420–6300, 6032–5783, 5633–5540, 5290–5200, and 5005–4955 A.[4] These bands give the yellow-green color to the negative glow in a Geissler tube.

Extending from 4870A in the visible to about 2200A and perhaps beyond are other bands which will be designated as belonging to the "first negative system." Johnson[4] has photographed these bands in a Geissler-tube discharge and has given an analysis of the system.[5] The wave-lengths of these bands as noted by the writer agreed with those given by Johnson. The appearance of the first negative bands under the experimental conditions obtaining here

[1] Sommerfeld, *Atombau und Spektrallinien*, chap. vii.

[2] Duffendack, *Astrophysical Journal*, **61**, 209, 1925.

[3] The names "first" and "second" negative band systems were assigned to these particular bands because they appear in the order given, as the voltage of the tube is increased, and they both belong to the ionized molecule, as will be shown. The term "negative bands" has been generally applied to those which are more intense in the negative glow of a Geissler-tube discharge than in the positive column. The term "positive bands" is applied to those which are more intense in the positive column.

[4] R. C. Johnson, *Proceedings of the Royal Society of London*, A, **105**, 683, 1924.

[5] Professor Birge, of the University of California, states in a letter that he has assigned new quantum numbers, n' and n'', to these bands by making the substitutions, $n'=33-n$, and $n''=55-p$, n and p being the quantum numbers of Johnson.

confirms his conclusion that these bands are not due to ozone. In the experiments any ozone would have been taken up immediately by the liquid air and the gas would have been "cleaned up" rapidly. No such clean-up was noted until at about 70 volts, and the bands appeared first at 19.2 volts.

The so-called "water-vapor" band[1] at 3064A appeared alone on some plates taken at about 18 volts. This band has been studied by several others, who have concluded that it is due to an *OH* molecule.[2] In the work of the writer the use of liquid air on the vacuum system eliminated the possibility of the presence of a large amount of water vapor. It is possible that a small amount could have been produced by vapors from the cement used in sealing on the quartz plate. This, of course, would not have been taken up by the liquid air until it had diffused to the trap.

The first negative bands appear on one set of plates at about 19.2 volts. At 18.2 volts, only the water-vapor band and the line 4368 are visible. At 19.2 volts, the band heads at 4352–4314, 4116–4063, 3958–3962, and 3734–3705A appear faintly. At 20.2 volts, the bands are developed to about 2300A. Since the first negative bands appear earliest at 19.2 volts, this value is taken to represent an energy-level for the initial electronic state of these bands. There is no trace of the second negative bands on one plate at 20.1 volts, but they are quite plain at 21.1 volts. On another plate the bands are visible at 21.8 volts, but not at 20.8. We may thus take 21 volts as the critical value for these bands. No changes in the bands occur at the highest voltages, but they become relatively weaker than the lines, due to an increase in the percentage of dissociation, and, of course, the enhanced lines appear on the plates.

We have just obtained two energy-levels for electronic states of the molecule. The vibrational levels for each state will be in steps slightly above the corresponding electronic levels, and those for the rotational states will be grouped around the vibrational levels. Only the electronic levels, however, will be considered. If a molecule in the 21-volt state reverts to the 19.2-volt state with the emission

[1] Steubing, *Annalen der Physik*, **33**, 553, 1910.

[2] Watson, *Physical Review*, **23**, 768, 1924. Dr. Barker, of the University of Michigan, also arrived at the same conclusion.

of radiation, the energy emitted will be that corresponding to the potential difference 1.8 volts, and the wave-length will be about 6850A, as given by the equation $\lambda = hc/Ve$. One of the second negative bands emitted during such a transition of the molecule has been observed at 6853–6567.

We will now consider the process of finding the final energy-level of the first negative bands from the initial level and the wave-length of the radiation. The earliest of the first negative bands to appear toward the violet is the one at 3734–3705. The voltage difference between the initial and final states calculated from the wave-length is about 3.3 volts, and the final level is thus 15.9 volts. In the experiments discussed in the first part of the paper an ionization potential was found at 16.1 volts. Since an accuracy to a tenth of a volt is beyond realization in this type of measurements, the agreement is rather good. Because of this agreement, we now interpret the ionization potential of 16.1 volts, previously obtained, as being an energy-level for the molecule, and in particular the lowest level for the ionized molecule. Photographs taken at voltages below 16.1 volts with tubes of two and of three elements showed no bands or lines in the region studied. All radiation produced at voltages below this value must be in other spectral regions.

It is interesting to note that only every other one of the bands found by Runge and Grotrian[1] were seen by Johnson and the writer, the others being missing. The ones found were those at 2983, 3233, 3516, 3841A, and the missing ones were those at 3104, 3370, and 3673A. It is possible that the high voltage and higher pressure excitation of Runge and Grotrian favored the development of these particular bands.

Interpretation of critical potentials.—Three energy-levels for the band spectra 21, 19.2, and 16.1 volts, have been found. This last potential has been interpreted as the ionization potential for the oxygen molecule. The value of 19.5 volts has been obtained for the potential at which dissociation of the molecule and ionization of an atom occur as the result of a single electronic encounter. A potential of 50 volts is sufficient for the production of doubly ionized oxygen atoms. My value for the ionization potential is in agreement with

[1] *Physica*, **1**, 254, 1921.

that of Mohler and Foote, the higher value of Mackay, and that of Smyth. Since we have reason to believe that the probability of dissociation of the molecule is very small below 19.5 volts, it will be necessary to interpret the experimentally determined critical potential at 8 or 9 volts as one for molecular radiation as well as for atomic radiation. We may then take this as an energy-level for the molecule.

The difference in the potentials for dissociation and ionization (19.5 volts) and for ionization of the atom (13.56 volts) gives a value of approximately 6 volts for the dissociation potential of the oxygen atom. The energy of dissociation calculated from this value is 138,000 calories per gram molecule.[1]

In closing the author wishes to acknowledge his indebtedness to Dr. O. S. Duffendack, of the University of Michigan, for his interest in the work and for the helpful suggestions regarding it.

DEPARTMENT OF PHYSICS
UNIVERSITY OF MICHIGAN
ANN ARBOR, MICH.
July 27, 1925

[1] See also Wulf, *Journal of the American Chemical Society*, **47**, 1945, 1925.

ON THE ORIGIN OF BRIGHT LINES IN STELLAR SPECTRA[1]

By SVEIN ROSSELAND[2]

ABSTRACT

Origin of emission lines in stellar spectra.—The discussion distinguishes four principal cases:

1. A star in hydrostatic and radiative equilibrium in which the emission of radiation from any small portion of matter corresponds to the case of true temperature equilibrium would show absorption lines over the main stellar disk, while the atmosphere projecting outside the disk would show emission lines.

2. Owing to general atomic properties, the thermal state of matter assumed in (1) can only be realized in nature, provided the pressure and temperature exceed certain limits. Below these limits, i.e., in the outer parts of the stellar atmospheres, fluorescence comes into play, manifesting itself all over the stellar disk, and it is suggested that bright line emission in early-type stars is due to such causes.

3. Any kind of penetrating radiation (i.e., in the nature of α-, β-, or γ-rays) entering the atmosphere must produce bright-line radiation, as shown, for instance, in the case of the terrestrial aurora borealis.

4. If the hydrodynamic state of the atmosphere is far from a state of hydrostatic equilibrium, local regions of high temperature may be formed in the atmosphere, which necessarily must give rise to bright-line emission.

It is tentatively suggested that bright lines in late-type spectra owe their origin principally to such causes as are enumerated under (3), or possibly (4); the strong excitation of the gases concerned would seem to exclude (2), while (1) is excluded by the low density of the stars in question.

The radiation emitted from a star is generally resolvable into a continuous spectrum on which is superposed selective absorption in lines and bands. In a minor number of objects, however, bright lines are found to be superposed on the general continuous spectrum. The great scarcity of these objects shows that the conditions underlying the formation of bright lines are such as are not generally present in the atmosphere of an average star.

1. *Classification of bright-line stars.*—From the point of view of spectral classification, the objects showing bright lines fall essentially into two distinct classes, one at each end of the spectroscopic sequence. The first class contains stars of types O and B only. The early subdivisions of O-type stars show the Balmer series of hydrogen and the spectrum of ionized helium in pure emission. In the later subclasses the high frequency members of the Balmer series weaken and change into absorption. The resulting mixture of ab-

[1] *Contributions from Mount Wilson Observatory*, No. 309.

[2] Fellow of the International Education Board, 1924–26.

sorption and emission lines within the same spectral series is an essential characteristic for the class in question, as it is found among all bright-line B-stars hitherto observed.[1] To this class bright-lines nebulae seem also to belong, since, with very few exceptions, they are all associated with B and O stars with which they probably are physically connected.

At the other end of the spectral sequence we find bright-line objects very different from those considered above. With few exceptions they are all giant stars showing variability in light in long periods. The most notable exceptions[2] are some dwarf stars which show bright hydrogen lines, and vary little, if at all, in brightness. Although the spectra of long period variables, as far as bright lines are concerned, are essentially characterized by the presence of hydrogen lines, other elements are represented by bright lines which sometimes are rather conspicuous. The wave-lengths of the bright lines show displacements to the violet, relative to the absorption lines, which vary in amount during the period of light variation and indicate that the radiating matter is constantly flowing out from the interior of the stars. The intensity distribution among the lines of the Balmer series is such that $H\delta$ is the strongest line, the intensities of the other lines decreasing to both sides, so that $H\gamma$ is fainter than $H\delta$ but stronger than $H\beta$, etc. It is significant, however, that in a small group of stars of very high luminosity, the intensity of the hydrogen lines decreases more or less regularly from $H\alpha$ to $H\beta$, etc. Bright lines are also essential characteristics of the spectra of variables of classes N, R, and S, and of irregular variables of late types. While hydrogen is still the dominating feature, other elements are invariably present, especially iron and titanium. The gap between these two classes is not complete, as bright lines are found occasionally in the spectra of Cepheids and irregular variables of spectral classes F, G, and K, the stars in question all being of exceptionally high luminosity.

[1] A complete survey of the bright-line B-stars is given by Merrill, Humason, and Burwell, *Contributions from Mount Wilson Observatory*, No. 294; *Astrophysical Journal*, **61**, 389, 1925.

[2] Adams and Joy: *Publications of the Astronomical Society of the Pacific*, **32**, 158, 1920; **34**, 174 and 175, 1922.

2. *General theoretical considerations.*—In order to give precision to the conceptions which will be used, it is convenient to recall briefly some definitions and formulae of the theory of radiative equilibrium.

Denote in a general way the radiation emitted per second from a certain volume element δV (situated at the point x, y, z), directed within an element of solid angle δw and contained in the frequency interval $d\nu$ by the expression

$$E_\nu \, d\nu \, \delta w \, \delta V,$$

where E_ν is a function of x, y, z, ν and the direction cosines of the positive axis of δw. Denote by $I_\nu \, d\nu \, \delta w \, \delta\sigma$ the radiation in the frequency interval $d\nu$ which, per second, flows normally through the surface element $\delta\sigma$ and is contained in the solid angle δw. Considering the difference in flow in a distance ds along the direction of δw, we see without difficulty that, if there is to be no change of state in the course of time, I_ν and E_ν must be connected through the differential relation[1]

$$\frac{\partial I_\nu}{\partial s} + x_\nu I_\nu = E_\nu . \tag{1}$$

The letter x_ν stands for the sum of the coefficients of absorption and scattering. We now introduce the notion of the optical thickness of a layer extending from a point A to a point B by the expression

$$(\tau_\nu)_{AB} = \int_A^B x_\nu \, ds \tag{2}$$

the integral being taken along the straight line joining the two points in question. The left-hand side of (1) may now be written in the form

$$e^{-\tau_\nu} \frac{\partial}{\partial s} (I_\nu e^{\tau_\nu})$$

[1] If the state of the system is variable in time t, an additional term $(1/c)(\partial I_\nu/\partial t)$ must be added to the left-hand side of (1), c being the velocity of light.

where the point A is assumed to be fixed while B is variable. Introducing τ_ν as a variable, instead of s, and integrating from A to C, we obtain from equation (1) the equivalent form

$$(I_\nu)_C=(I_\nu)_A\, e^{-(\tau_\nu)_{AC}}+\int_0^{(\tau_\nu)_{AC}}\frac{E_\nu}{x_\nu}e^{-z}dz\,;\quad z=(\tau_\nu)_{BC}. \tag{3}$$

The path of integration is in the direction opposite to the direction of I_ν. As E_ν will, in general, depend on I_ν, (3) is an integral equation for the determination of I_ν, which, for special cases, will reduce to a linear integral equation of the Fredholm type.

The simplest case is met with when the function E_ν corresponds to true temperature equilibrium. We have then by Kirchoff's law that $4\pi E_\nu/c$ is equal to the product of the absorption coefficient and the energy density of black body radiation B_ν. Thus

$$\frac{4\pi}{c}E_\nu=x_\nu B_\nu\,;\qquad B_\nu=\frac{8\pi h\nu^3}{c^3}\left(e^{\frac{h\nu}{kT}}-1\right)^{-1} \tag{4}$$

where h, k, c, and T have their usual meaning. In this case we shall say that the system is in a quasi-thermal state, the difference from true temperature equilibrium consisting in the fact that the temperature T may vary from point to point, as a consequence of which there will be a net flux of radiant energy at each point, and the energy density of radiation will not be equal to B_ν. The conditions for the possible appearance of bright lines in the spectrum of the radiation emitted from a star in a quasi-thermal state are simple, as they depend solely upon the optical thickness of the star for general absorption, averaged in some way over the stellar disk. In order to see how this result emerges from the formula (3), we extend the integration from a point outside the star along a straight line so far into the star that the optical thickness of the intervening layer τ_ν is very large compared with unity. The last term in (3) is then negligible, and, on using (4), the first term may be written in the form

$$I_\nu=\frac{c}{4\pi}\bar{B}_\nu'(1-e^{-\tau_\nu})\approx\frac{c}{4\pi}\bar{B}_\nu \tag{5}$$

where $\bar{B}_\nu$ is the average value of B_ν along the path of integration, the weight function being $e^{-\tau_\nu}$. If the star is in a steady state, the temperature must of necessity increase on passing from the surface toward the interior, and, as B_ν increases rapidly with increasing temperature, it follows that the mean value of B_ν appearing in the foregoing formula will coincide approximately with the actual value of B_ν at the point below the surface where the optical thickness becomes comparable with unity.[1] The larger the absorption coefficient, the smaller is the distance below the surface at which a certain value of τ_ν will be attained, and, by (5), the smaller the intensity of the emergent radiation. Thus the spectral lines will show up in absorption at this point on the stellar disk.

At the limb of the star it may happen, however, that the total optical thickness in the line of sight is not large compared to unity, as a consequence of which we obtain, instead of (5), the formula

$$I_\nu = \frac{c}{4\pi} \bar{B}_\nu (1 - e^{-\tau_\nu}) \infty \frac{c}{4\pi} \tau_\nu \bar{B}_\nu . \tag{6}$$

This just reverses the foregoing result, since I_ν now increases with increasing values of the absorption coefficient, and, in consequence, the spectral lines will be bright. Defining the atmosphere of a star as the boundary region, the optical thickness of which for the main spectral region is comparable to unity, we understand that the main stellar disk will show a continuous spectrum with absorption lines, while the atmosphere, as seen at the limb, will show a spectrum principally of emission lines. Hence the simple result that the condition for the appearance of emission lines in the integrated spectrum of the radiation emitted from a gaseous star in a quasi-thermal state is determined by the relative extension of its atmosphere. If this atmosphere outgrows its nuclear star completely, we have the limiting case of a quasi-thermal nebula showing a spectrum of emission lines.

The conditions necessary for the maintenance of a quasi-thermal state may be violated in three different ways: (1) The star in ques-

[1] The validity of this argument is limited by the implicit requirement that B_ν must increase at a slower rate than e^{τ_ν}, otherwise the integral would not converge, the intensity of emergent radiation being infinitely large.

tion may not be in a steady state: local variations in temperature may exist, producing, for instance, high temperature faculae and prominences. (2) The material may emit radiation at a rate which is independent of its temperature or of the intensity of the radiation sifting through the medium. Such is actually the case for the radiation emitted from radioactive substances. It does not matter much whether this radiation is ultimately corpuscular or electromagnetic in nature, as both would tend to set up a state of things which cannot be described by assigning to the substance any single temperature. (3) The radiation absorbed by the substance may not be changed into energy of true temperature radiation before being reradiated. When this is the case, the transformation of the radiation absorbed will depend essentially upon the scattering power of the substance. Such is the case for the phenomena of scattering, resonance, and fluorescent radiation. In any of these cases the conditions for the appearance of emission or absorption lines are different from those applying to the quasi-thermal case. We shall now discuss the third alternative more closely.

3. *Fluorescence phenomena in stellar atmospheres.*—A little reflection will show that in the outer atmosphere of a star conditions will be such as to favor the formation of emission lines by fluorescence. In order to elucidate this point it is necessary to consider the physical processes underlying fluorescence phenomena in general. As we are dealing with gases only, and in most cases with monatomic gases, the question is much simplified, as it appears to be an established fact that, for a given temperature, the appearance of fluorescent radiation is a question of pressure only. The simplest instance of such processes is provided by mercury vapor, which, at pressures above a certain limit, absorbs radiation of wave-length 2536.7 A very strongly, the absorbed energy being converted directly into translational energy of temperature-motion of the atoms. For pressures below the critical limit, however, the absorbed radiation is largely re-emitted in all directions with unchanged wave-length. This influence of pressure on fluorescence phenomena is undoubtedly due to molecular collisions (of the so-called "second kind"). During the absorption process the atoms are transferred to some higher quantum state from which, after the lapse of some time, they return

to the normal state under emission of radiation. Collisions of such excited atoms with other atoms of the gas may have the result that the absorbed energy is, in the collision process, changed into energy of translation of the colliding atoms. The critical pressure must correspond to the case when the mean time between two successive collisions of an atom is of the same order of magnitude as the mean time interval in which the atom will remain in the quantum state from which (or in which) the emission process in question takes place.[1]

In the special case considered above the scattered radiation has the same wave-length as the incident radiation, because no free transition of the atom is possible except a reversion to the state from which it started. In the general case the scattered radiation will, by Stokes's law, be distributed over longer wave-lengths than the incident radiation, the law of distribution being determined by the energy distribution in the spectrum of the incident radiation and by the internal structure of the atoms in question, but not by the temperature of the substance as it may be calculated from the equation of state. In experiments on the extinction of resonance radiation in mercury the critical pressure has been found to be about one one-hundredth of an atmosphere, which indicates a mean life-period of the excited atom of about 10^{-7} to 10^{-8} seconds. This, again, is about what might be expected from theory.

Consider now the spectrum of the radiation received from a star by an observer situated at a distance r from its center, r being assumed to be larger than the radius of the star. Apart from the effect of darkening at the limb and the effect of absorption and emission lines, this spectrum, in the majority of cases, will show a fair resemblance to the spectrum of black body radiation, provided it is cut down by a factor W which is the same for all wave-lengths. The quantity $4\pi W$ is equal to the solid angle subtended by the star as seen by the observer. The expression of the energy density of radiation in the frequency interval ν to $\nu+d\nu$ at the point in question is thus

$$\rho(\nu)d\nu = WB_\nu\, d\nu, \tag{7}$$

[1] For literature on this subject, cf. R. W. Wood, *Philosophical Magazine*, **23**, 689, 1912; J. Franck, *Zeitschrift für Physik*, **9**, 259, 1922; G. Cario, *ibid.*, **10**, 185, 1922; G. Cario and J. Franck, *ibid.*, **17**, 202, 1923.

the temperature involved in the expression for B_ν being the effective temperature of the star. The radiant field in question does not correspond to any kind of temperature radiation, as it has virtually a temperature associated with each infinitely small frequency interval. In the ultra-violet region ($h\nu$ large compared to kT) the energy density corresponds asymptotically to the temperature

$$T\left(1-\frac{kT}{h\nu}\log W\right)^{-1} \tag{8}$$

which for very large frequencies approaches indefinitely near to the effective temperature T of the star. In the region of long wave-lengths ($h\nu$ small compared to kT) we may write asymptotically

$$\rho(\nu)=\frac{8\pi\nu^2}{c^3}kWT\,. \tag{9}$$

In this region the radiation has the virtual temperature WT, which is inversely proportional to the square of the distance from the center of the star, and for very large distances may, therefore, be only a minute fraction of the virtual temperature in the region of short wave-lengths.

A stratum of gaseous matter in radiative equilibrium with radiation from the star, placed at the point in question, will take up a temperature t which satisfies the inequality $T>t>WT$, provided, of course, that the state of the gas is quasi-thermal.[1] This gas is, therefore, "cold" relative to the radiation of the region of short wave-lengths, and "warm" compared to that of the long waves. If the pressure of the gas falls considerably below the critical value, there seems to be no reason why the temperature (which now can be obtained only from the equation of state) should increase appreciably, although there is now no tendency for the gas to take up a quasi-thermal state, even approximately. The degree of departure from such a state will depend essentially on the distance of the stratum of gas from the photospheric layer of the star. In particular,

[1] C. Fabry, *Journal de Physique*, **6**, 207, 1916, and E. A. Milne, *Monthly Notices of the Royal Astronomical Society*, **82**, 368, 1922.

the transformation of the radiation which is absorbed in a line spectrum will consist in the re-emission of another line spectrum, essentially of lower frequencies. There is apparently nothing in the trend of the foregoing argument which will be altered if the gas forms a part of the upper atmosphere of a star, and hence we are led to the following conclusion: In the atmosphere of a star, a closed surface may be drawn such that the matter outside this surface cannot be in a quasi-thermal state. In this part of the atmosphere emission of bright-line radiation will be likely to take place, the intensity depending, however, on a large number of different factors. In view of the rather large values of the critical pressure found in terrestrial experiments, it may further be concluded that the hypothetical bright-line emission from a quasi-thermal atmosphere or even from a nebula, as considered in section 2, is not realized in nature.

Let us now consider how far actually observed bright-line emission can be ascribed to fluorescence phenomena of the kind discussed above. It will then appear that the question is likely to be answered in the affirmative, as far as early-type stars are concerned. The most weighty argument in favor of such a hypothesis is afforded by the fact that in these stars the intensity of emission lines within the same spectral series is greatest for the members of long wave-length, while those of short wave-length most frequently show as absorption lines. This fact immediately reminds us of Stokes's fluorescence law. Moreover, the effective temperature of these stars is so large that the prominence of hydrogen and helium as fluorescent substances does not present any difficulty at all. The extension of the atmospheres which is necessary for the development of fluorescent radiation to the observed extent must probably be very great.

4. *Mathematical theory.*—The problem of determining the state of a fluorescent atmosphere in radiative equilibrium is beset with many difficulties, and only the outlines of the theory will be discussed, detailed applications being reserved for later papers. The equation of radiative transfer is still given by (1) provided E_ν is properly specified as a function of I_ν and the density of the gas. The function E_ν will be different for different kinds of emission processes. Consider the case of pure scattering. The scattering coefficient may be denoted by σ_ν. The radiation scattered per unit time from a vol-

ume element δV in the frequency interval $d\nu$ and within an elementary solid angle δw is

$$S_\nu \delta w \delta V d\nu = \delta w \delta V d\nu \int \sigma'_\nu I'_\nu dw'$$

or

$$S_\nu = \int \sigma'_\nu I'_\nu dw' \,. \tag{10}$$

The integral in (10) is to be extended to 4π of solid angle. The quantity S_ν is the contribution to E_ν arising from pure scattering without change of wave-length. The coefficient σ_ν is, according to ordinary electromagnetic theory, proportional to $1+\cos^2 i$ where i is the angle between the direction of the scattered radiation and the direction of the incident radiation. It is, moreover, a function of the state of excitation of the substance as, for instance, no selective scattering of the hydrogen Balmer lines will take place if all the atoms reside in the normal state. In the case of purely radiative equilibrium (collisions between atoms having a negligible effect on the state of excitation) the excitation will be effected by absorption of radiation only, and thus σ_ν will involve I_ν in some form or other.

Consider next the case of spontaneous emission. Let the frequency interval ν to $\nu+d\nu$ lie within the region of a spectral line of mean frequency ν_0 corresponding to the free transition $i \to k$ of the atom. Let a_{ik} be the probability of this particular transition and n_i the number of atoms per unit volume in the i-state. The intensity of the spontaneous radiation must be equally distributed in all directions and the analogue of S_ν may be written directly in the form

$$R_\nu = \frac{h\nu}{4\pi} n_i a_{ik} f_{ik}(\nu) \tag{11}$$

The factor f_{ik} has been introduced to account for the intensity distribution within the line. It must, according to Bohr's frequency relation, satisfy the equation

$$\int_0^\infty f_{ik}(\nu) h\nu d\nu = h\nu_0 \tag{12}$$

In view of the fact that f_{ik} decreases very rapidly with increasing values of $|\nu-\nu_0|$, we must have, approximately,

$$\int_0^\infty f_{ik}(\nu) d\nu = 1$$

The processes discussed thus far give for E_ν the expression

$$E_\nu = \frac{h\nu}{4\pi} n_i a_{ik} f_{ik}(\nu) + \int \sigma'_\nu I'_\nu dw' \tag{13}$$

As the scattering coefficient σ_ν must be proportional to n_i the functional form of E_ν, as regards its dependence on I_ν, is completely specified when n_i is given as a function of I_ν. The formal solution of this latter problem is very simple. In fact, in a steady state, the number of atoms leaving a particular stationary state i in any given interval of time must be equal to the number of atoms entering this state in the same time interval. Thus we have

$$n_i(a_{i1}+a_{i2}+\ \ldots\ .\ a_{ir}) = n_1 a_{1i} + n_2 a_{2i} + \ \ldots\ .\ n_r a_{ri}\ ,$$
$$i = 1.2\ \ldots\ .\ r \tag{14}$$

No transition coefficient a_{ii} occurs in this equation, and the introduction of one would have no meaning, as it would appear on both sides and drop out identically. We may, therefore, use the notation

$$a_{ii} = -\sum_{k=1}^{r} a_{ik}\ ; \qquad k \gtrless i\ ; \qquad i = 1.2\ \ldots\ .\ r \tag{15}$$

and write (14) in the form

$$\sum_{i=1}^{r} n_i a_{ik} = 0\ ; \qquad k = 1.2\ \ldots\ .\ r \tag{16}$$

r being the total number of physically distinct quantum states of the system. The determinant

$$A = |a_{ik}| \tag{17}$$

is zero in virtue of the relations (15).[1] Denoting the algebraic complement of a_{ik} in the determinant (17) by A_{ik}, we find the general solution of (16) to be

$$n_i = \lambda_m A_{im}\ ; \qquad i = 1.2\ \ldots\ .\ r \tag{18}$$

where λ_m is independent of the index i. The transition coefficients a_{ik} are, according to the well-known theory of Einstein, linear func-

[1] Pascal, *Determinanten* (Leipzig, 1900), p. 10.

tions of the energy density of the radiation field $\rho(\nu)$ for the frequency which corresponds to the transition in question. Now

$$\rho(\nu)=\frac{1}{c}\int I_\nu dw\,, \tag{19}$$

the integral being extended to 4π of solid angle. Thus n_i, by (18) and (19), becomes a function of the intensity of the radiation field for a large number of different frequency intervals distributed throughout the entire spectrum.

5. *Thermal equilibrium.*—The foregoing theory applies to any kind of radiative equilibrium. The equilibrium will be said to be "thermal" if there is an independent balance of transition processes between any two arbitrarily selected states, such that

$$n_i a_{ik}=n_k a_{ki} \tag{20}$$

The restrictions involved by these relations concerning the agencies underlying the transition processes are perhaps better elucidated by the statement: In thermal equilibrium the probability that an atom shall complete a closed route of transitions

$$i_1\rightarrow i_2\rightarrow \ldots . \ i_n\rightarrow i_1$$

is equal to the probability that it shall complete the reverse route

$$i_1\rightarrow i_n\rightarrow \ldots . \ \rightarrow i_2\rightarrow i_1$$

in the same time. As the transition probabilities are all independent, the statement in question takes the form

$$a_{i_1i_2}a_{i_2i_3}\ldots . \ a_{i_ni_1}=a_{i_1i_n}\ a_{i_ni_{n-1}}\ldots . \ a_{i_2i_1} \tag{21}$$

This statement may suitably be called the "principle of reversibility." Equation (21) follows from (20) by writing down the series of equations

$$n_{i_1}a_{i_1i_2}=n_{i_2}a_{i_2i_1}$$

$$n_{i_2}a_{i_2i_3}=n_{i_3}a_{i_3i_2}$$

$$\cdot \qquad\qquad \cdot$$

$$\cdot \qquad\qquad \cdot$$

$$n_{i_n}a_{i_ni_1}=n_{i_1}a_{i_1i_n}$$

and multiplying the corresponding members. All the n's drop out, and leave (21) as a result. Equations (21) are equivalent to (20).

If we knew the laws governing the transition processes, and thus the ratios of the different a's, which by (18) determine the equilibrium state, it would be possible to specify the state of the system in thermal equilibrium on the basis of the foregoing theory. The principle of reversibility alone is not sufficient for this purpose, and additional hypotheses of a mechanical and electrodynamical kind are necessary.[1] In the present state of science it seems, however, more satisfactory to appeal to the general statistical theory of thermal equilibrium in which an explicit reference to the transition coefficients is avoided. Taking thus the ratios n_i/n_k for granted, we find it possible, by (20), to determine new relations between the transition coefficients. The result of this procedure for the present case is well known. By Boltzmann's principle

$$n_i/n_k = p_i/p_k\, e^{-\frac{\chi_i-\chi_k}{kT}} \tag{22}$$

where p_i is the weight and χ_i the energy of the i-state, and p_k, χ_k are the same quantities for the k-state. For transitions between quantized states the transition coefficients may be shown to satisfy the relations

$$a_{ik} = a_{ik}(1+\bar{\rho}_\nu)\ ; \qquad \chi_i > \chi_k \tag{23}$$

$$a_{ki} = \frac{p_i}{p_k}\, a_{ik}\bar{\rho}_\nu \tag{24}$$

where a_{ik} is the probability of spontaneous transition $i \to k$, while $\bar{\rho}_\nu$ is the energy density of radiation for the frequency

$$\nu = (\chi_i - \chi_k)/h$$

[1] Cf. A. S. Eddington, *Philosophical Magazine*, 50, 803, 1925, where this question is discussed in detail. It appears that the distribution of energy with frequency (Planck's formula) may be obtained on the basis of the reversibility principle, provided Wien's displacement law, the $h\nu$-relation, and the linear relationship between the transition coefficients and the energy density of radiation for the particular frequency in question are taken for granted. Some approach toward a derivation of Boltzmann's formula for the distribution of atoms over the different quantum states can also be made on the same basis.

divided by $8\pi h\nu^3 c^{-3}$. The relations (23) and (24) are generally valid, whether the system is in thermal equilibrium or not.

6. *Equilibrium in reduced thermal radiation.*—A few simple applications of the foregoing theory for the case that the energy density of radiation is given by (7) will now be shown. The simplest case, which is not trivial, is offered by atoms possessing three possible states only, and this gives us a first idea about the nature of the distribution law in a fluorescent atmosphere. We assume the states to be enumerated in such a way that $\chi_1 < \chi_2 < \chi_3$. The sub-determinants entering (18) may be taken to be

$$A_{11} = a_{21}(a_{31}+a_{32})+a_{23}\,a_{31} \tag{25}$$

$$A_{21} = a_{12}(a_{31}+a_{32})+a_{13}\,a_{32} \tag{26}$$

$$A_{31} = a_{13}(a_{21}+a_{23})+a_{12}\,a_{23} \tag{27}$$

From these expressions we obtain

$$-(a_{31}A_{31}-a_{13}A_{11}) = a_{32}A_{31}-a_{23}A_{21} = a_{21}A_{21}-a_{12}A_{11} = \Delta$$

where

$$\Delta = a_{13}\,a_{32}\,a_{21}-a_{12}\,a_{23}\,a_{31} \tag{28}$$

Using expressions (23), (24), and (7), we find

$$a_{12}a_{23}a_{31} = y\,a_{13}a_{32}a_{21}\ ; \tag{29}$$

Where

$$y = \frac{W^2 F_{31}}{F_{32}F_{21}}\ ; \qquad F_{ik} = 1-(1-W)e^{-\frac{\chi_i-\chi_k}{kT}}$$

When W is less than unity the quantity y is positive and less than unity, and equation (29) shows that it is easier for the atom to complete the circuit over all states in the direction $1\to3\to2\to1$ (one absorption-act and two emissions) than in the direction $1\to2\to3\to1$ (two absorptions and one emission). The effect of the atoms on the stream of radiation will, therefore, be to create one absorption line, corresponding to the transition $1\to3$, and two emission lines, corresponding to the transitions $3\to2$, $2\to1$. This result illustrates clearly the general process of generation of emission lines by fluorescence.

While the particular values of the spontaneous transition coefficients do not affect the law of distribution of atoms over the different quantum states for the case of thermodynamic equilibrium, they will do so for any other kind of equilibrium. As an extreme example consider the case when the 2-state is "metastable," i.e., $a_{12}=a_{21}=0$. We then obtain from (18) and (25) and (26)

$$n_2/n_1=\frac{p_2}{p_1}e^{-\frac{\chi_2-\chi_1}{kT}}\frac{1-(1-W)\,e^{-\frac{\chi_3-\chi_2}{kT}}}{1-(1-W)\,e^{-\frac{\chi_3-\chi_1}{kT}}} \tag{30}$$

This expression shows that the excitation to the metastable state may be approximately the same as in the case of thermodynamic equilibrium ($W=1$) at the temperature T, even if W is very small. This result is of considerable interest for the theory of gaseous nebulae.[1]

The theory of radiative equilibrium developed in section 4 is valid even if ionization takes place. As a first approximation we shall consider the case of a single element, and take account of one quantum state only. Let the index 1 refer to the atom in an unionized state and 2 to the ionized state. The ionization formula takes the form

$$n_2/n_1=a_{12}/a_{21} \tag{31}$$

The transition coefficients a_{12} and a_{21} can be calculated as soon as the atomic absorption coefficient x_ν is known as well as the velocity distribution of the free electrons. To save repetition from earlier work[2] I give directly the expressions for the a's

$$a_{12}=\int_0^\infty x_\nu c\rho(\nu)d\nu/h\nu \tag{32}$$

$$a_{21}=\frac{2\sigma_{12}n_e}{hm^2c^2}\int_0^\infty x_\nu v^2(1+\bar{\rho}_\nu)F(v)v^{-1}\frac{dv}{d\nu}\,d\nu \tag{33}$$

[1] *Nature*, **114**, 859, 1924.

[2] Cf. especially E. A. Milne, *Philosophical Magazine*, **47**, 209, 1924.

where $n_e F(v)dv$ is the number of free electrons per unit volume in the velocity interval v to $v+dv$ and σ_{12} the a priori probability of the atom with the electron bound relative to the a priori probability of the atom when the electron is missing.

As an illustration consider the explicit ionization formula for the case that the velocities of the electrons are distributed according to Maxwell's law. The energy density of radiation is given by (7), and, as an approximation, we assume that x_ν is zero in the interval $\nu=0$ to $\nu=\nu_m$ and proportional to ν^{-2} for higher frequencies. Reducing expressions (32) and (33) according to these assumptions, we find

$$\frac{n_2 n_e}{n_1}=\frac{WT}{T_0}\frac{(4\pi mkT_0)^{\frac{3}{2}}}{\sigma_{12}}\frac{\log\left(1-e^{-\frac{h\nu_m}{kT}}\right)}{\psi} \tag{34}$$

where

$$\psi=\int_{\nu_m}^{\infty}\left(1+\frac{W}{e^{\frac{h\nu}{kT}}-1}\right)e^{\frac{h\nu_m-h\nu}{kT_0}}\frac{h\,dv}{kT_0} \tag{35}$$

and where T_0 is the temperature of the electrons. Formula (34) should be of value in any discussion of the ionization of a gaseous nebula excited by radiation from a distant star. The special assumptions introduced concerning the velocity distribution of the electrons and the functional form of the absorption coefficient cannot have influenced the result very much.

7. *Bright lines in spectra of the late type.*—Turning to the stars of late type, we find that the problem of accounting for bright-line radiation assumes a much more complicated aspect. The surprising thing is not so much the appearance of bright lines as the fact that lines requiring such strong excitation in order to appear as the hydrogen lines should be so amply represented in the spectra of these low-temperature stars. The origin of this excitation appears to constitute the essential problem at issue, the solution of which must be known before any suggestion about the mechanism of the production of bright lines can be ventured.

The hydrogen lines appear as strong absorption lines in early-type stars, and decrease in intensity all the way through the spectral

sequence from class A to classes F, G, and K. There is a remarkable difference between giants and dwarf stars in that giants are likely to exhibit hydrogen absorption lines much more strongly than dwarfs of the same spectral type. This peculiarity is particularly prominent for late-type stars, many giant K and M stars exhibiting prominent absorption lines while dwarf stars of the same types generally show very faint hydrogen absorption.

The general decrease in intensity of hydrogen absorption lines in passing from spectra of early to those of late type is satisfactorily explained by ordinary thermodynamic theory. The different behavior of giants and dwarfs may be due partly to different thickness of atmospheres, and partly to a real difference in absolute concentration of hydrogen atoms in the stars of the two branches. Such a difference is to be expected if, in the stars, heavy elements are built up from hydrogen, provided, of course, that the evolutionary course of a star is in the direction

$$M_{giant} \rightarrow A \rightarrow B \rightarrow A \rightarrow M_{dwarf}$$

as is usually assumed.

When it is a question of accounting for the presence of hydrogen lines in giant M stars, thermodynamic theory seems to break down. This theory would predict that at a temperature of 2000° K. only one atom out of every 10^{24} would be able to absorb or emit the Balmer lines. This proportion is so small that the thickness of atmosphere required to produce detectable lines, for the probable values of the mean density of such stars and of the absorption coefficient for the lines in question, appears to be much larger than the dimensions of the stars. This result seemingly points to the conclusion that the excitation of hydrogen atoms, especially in the atmospheres of long-period variable stars, is not connected in a simple manner with the mean temperature of the photospheric layers. From the appearance of lines of elements other than hydrogen, little can be concluded, either for or against this inference. The lines in question are mostly such as appear in consequence of so small an excitation that temperature alone might account for the main part of this excitation, and any differentiating agency would be sufficient to

make them visible. They cannot, therefore, be invoked as evidence when it comes to a distinction between different modes of excitation.

Dropping the hypothesis that bright hydrogen lines in M stars are produced by pure fluorescence phenomena in a quiescent atmosphere, as assumed to be the case for early-type stars, we see that the discussion in section 2 leaves only two possibilities: (1) We are witnessing the effect of electric discharges, as would be the case if the atmospheres were the seat of emission of penetrating β or α particles; or (2) the atmospheres are the seat of violent and local variations in temperature.

We cannot decide definitely which of these possibilities is the more important. The fact that the lines appear to be rather narrow and to possess a peculiar kind of structure[1] suggests that the part of the atmosphere from which they are emitted is in a relatively steady state and not subject to violent internal motion changing rapidly with the time, as would be expected if (2) were the important factor. A closer study of the problem in hand, from the observational as well as the theoretical side, might lead to interesting consequences regarding the state of the stellar atmospheres and to a better understanding of long-period variation in general.

Any complete theory of bright lines in late-type stars must account for the abnormal distribution of intensity among the lines of the Balmer series. This question appears to involve complications, and no suggestion as to the solution will be advanced here. It may be mentioned, however, that this distribution of intensity differs widely from that to be expected on any approximate application of formulae based on the equations corresponding to temperature equilibrium. The relative intensities seem, therefore, to afford an independent argument in support of the conclusions that any explanation of these phenomena based on the ordinary theory of radiative equilibrium is bound to be inadequate, and that consequently we are induced to consider alternative possibilities, as has tentatively been done above.

MOUNT WILSON OBSERVATORY
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[1] W. W. Campbell, *Astrophysical Journal*, **9**, 31, 1899.

A SPIRAL NEBULA AS A STELLAR SYSTEM MESSIER 33[1]

By EDWIN HUBBLE

ABSTRACT

The spiral nebula Messier 33.—This object is the fainter of the two naked-eye spirals. Its great angular diameter and high degree of resolution, suggesting that it is one of the nearest objects of its kind, offer exceptional opportunities for detailed investigation.

Resolution.—Critical tests made with the 100-inch reflector, the highest resolving-power available, show no difference between the photographic images of the so-called condensations in Messier 33 and the images of ordinary galactic stars. Moreover, the application of ordinary methods of stellar research leads to consistent results, similar to those found in other systems of stars.

Variable stars.—Forty-two variable stars have been found in the spiral, in addition to the three previously known; thirty-five of these are typical Cepheids with periods ranging from thirteen to seventy days, and photographic magnitudes at maxima, from 18.0 to 19.1. Of the remaining ten variables, four are irregular, one is probably an eclipsing binary, while the types of five are as yet undetermined.

Novae.—Two novae have been observed, both apparently on the declining branch of their light curves. The maximum observed photographic magnitudes, 17.25 and 17.9, respectively, were brighter than that attained by any of the Cepheids.

Nebulae.—Several patches of diffuse emission nebulosity with blue stars involved are found in the arms of the spiral. The relation between the extent of the nebulosity and the magnitudes of the stars is similar to that in the galactic system. It is inferred that the spectral types of the involved stars are Oe5 and Bo.

Star colors.—A comparison of photovisual with shorter direct exposures indicates that the brightest stars in the nebula are white or blue. No accurate calibration of the photovisual plates has been possible, but an approximate reduction curve was constructed from well-determined photographic magnitudes, corrected by color-indices corresponding to known or probable spectral types of a few exceptional stars.

Star counts.—The numbers of stars in successive intervals of photographic magnitude down to 19.2 have been counted over an area containing about one-fifth of all of the stars in the nebula. Stars begin to appear in the nebula at about 15.5. The form of the luminosity function is similar to the bright end of Kapteyn's function for stars in the vicinity of the sun.

Distance.—The period-luminosity relation is conspicuous among the thirty-five Cepheids and indicates a distance about 8.1 times that of the small Magellanic Cloud. Using Shapley's value for the latter, the distance of the spiral is about 263,000 parsecs. $m-M=22.1$. The order of this distance is supported by the independent evidence of the novae, the blue giants involved in diffuse emission nebulosity, and the form of the luminosity function for the brighter stars.

Dimensions.—The linear scale corresponding to this distance is $1''=1.27$ parsecs. The major diameter of the spiral is about 4600 parsecs, and the minor diameter probably a tenth of this amount. The total absolute magnitude is about -15.1. The dimensions and the luminosity are comparable with those for the Magellanic Clouds.

The nature of nebulae and their place in the scheme of the universe have been favorite subjects of controversy since the very dawn of telescopic observations. In these later days, when the non-galactic nebulae have been clearly differentiated from the diffuse and the

[1] *Contributions from the Mount Wilson Observatory*, No. 310.

planetary, the discussion has concentrated around the spectacular forms of the spirals. Some believe them to be subordinate members of the galactic system, and hence moderately small and relatively near; others consider them as comparable with the galactic system itself—island universes, scattered through the remote depths of space. The crucial point at issue is the order of their distance, and arguments concerning this quantity have been largely speculative.

The present contribution to the subject consists of observational data concerning the particular spiral, Messier 33. The data lead to a conception of the object as an isolated system of stars and nebulae, lying far outside the limits of the galactic system. To this extent the evidence favors the island-universe hypothesis, but, in respect to dimensions and luminosity, the spiral is more closely comparable with the Magellanic Clouds than with the galactic system itself.

The investigations described followed naturally upon the partial resolution of Messier 33 into swarms of actual stars, and ordinary methods of stellar research were employed throughout. The discovery of Cepheid variables[1] furnished a probably reliable criterion of absolute magnitude, and the order of the resulting distance was confirmed by other independent criteria. The same procedure, when applied to another spiral, Messier 31, led to similarly consistent results, strengthened in this case by the absolute-magnitude criterion for the numerous novae. The Andromeda spiral is the subject of a second paper, now in preparation, and the present discussion may be followed with this in mind.

RESOLUTION OF MESSIER 33

The familiar condensations in the arms of spirals have been constantly interpreted, both casually and analytically, as stars in the process of formation. Messier 33 is especially rich in condensations, and its great angular dimensions, exceeded only by those of Messier 31, suggest that it is one of the nearest of all the spirals. For these reasons, Messier 33 affords the best opportunity for investigating the optical, or rather the photographic, nature of the condensations.

[1] A preliminary notice of the discovery of Cepheid variables will be found in *Publications of the American Astronomical Society* (33d meeting), Jan., 1925; reprinted in *Observatory*, **48**, 139, 1925.

G. W. Ritchey, basing his judgment on the best material hitherto available, his own splendid photographs made with the 60-inch reflector, described them as soft, starlike condensations and designated them by the term "nebulous stars."[1]

The nebulous character, however, was inferred from the appearance of the images on long exposures made with fast reflectors centered on the nucleus of the spiral. The area of best definition was restricted to the central region where a considerable amount of unresolved nebulosity registered as a dense and fairly continuous background. In the outer regions of the nebula, where the background was relatively light, the various aberrations spread the images over considerable areas. Such conditions are not suitable for determining the character of the images. Considerations of this nature, supported by evidence from slitless spectrograms obtained with the 36-inch reflector at the Lick Observatory, led Knut Lundmark to the conclusion that the phenomena could be accounted for by assuming the spiral to consist of great numbers of ordinary stars with clusters of stars and some nebular material.[2]

[1] *Mt. Wilson Contr.*, No. 47; *Astrophysical Journal*, **32**, 26, 1910. Ritchey states: "All of these [spirals, including Messier 33] contain great numbers of soft starlike condensations which I shall call nebulous stars. They are probably stars in the process of formation." In Messier 33 he counted some 2400 "nebulous stars." This recalls the first description of the condensations in Messier 33 by Isaac Roberts based upon photographs made with his 20-inch reflector: "They are like nebulous stars in the midst of fainter nebulosity"; and again: "hundreds of stars having each a very small nucleus surrounded by fainter nebulosity" (*Photographs of Stars, Star Clusters and Nebulae*, **2**, 66, 1899).

[2] *Publications of the Astronomical Society of the Pacific*, **33**, 324, 1921. The statements are repeated in somewhat stronger terms in *Monthly Notices*, **85**, 890, 1925. It is, however, difficult to understand how the stellar nature of such faint light-sources can be "proved," as Lundmark claims, from spectrograms. To be assigned with certainty to the nebula, the objects must be of the order of the sixteenth photographic magnitude or fainter, and even prolonged exposures can do little more than distinguish between emission and continuous spectra. Lundmark found the latter and estimated them as approximately of solar type. This, of course, is just what is found for the small non-galactic nebulae scattered over the sky, and they, too, have often been called "nebulous stars." Moreover, there is considerable evidence, presented in the present paper, indicating that the brighter stars in Messier 33 are almost invariably white or blue. The discussion over the condensations in Messier 33, however, concerns the angular diameters of the small images, and, since they give continuous spectra, the spectrograms can have no advantage over direct photography, unless the ratio of camera to collimator is much larger than that ordinarily employed for faint objects.

The subject has been reinvestigated with the highest resolving-power available—the full aperture of the 100-inch reflector working under exceptionally fine conditions of mirror surface and seeing. The plates were centered on the outer regions of the arms as well as on the nucleus, and the exposures were short, not exceeding forty minutes, for the double purpose of avoiding the dense background of nebulosity and possible changes in observing conditions. On several plates, equal exposures were made to the nebula and to the neighboring Selected Area No. 45. This was primarily for the purpose of deriving magnitudes, but it also furnished an opportunity for comparing the images in the two regions.

A study of the plates clearly establishes the stellar appearance of the images of the condensations. They are as small and as sharp as those of ordinary stars of the same magnitude. The two most successful plates,[1] each of thirty minutes' exposure, centered on regions seven minutes of arc south and south-preceding the nucleus, show images with angular diameters smaller than any previously obtained on any astronomical photograph. The means of twenty of the smallest images are about 0".55 and 0".6, respectively. An enlargement from the central region of one of these photographs is shown in Plate XVI. An equal area in Selected Area No. 45, photographed on the same plate with the same exposure time, shows about sixty stars; hence most of the images in the illustration can definitely be assigned to the nebula.

Occasional patches, which show appreciable dimensions on plates made with smaller instruments, the 100-inch resolved into groups of stars, a few of which are surrounded by what appears to be true unresolved nebulosity. The knots listed as N.G.C. 588, 592, and 604 are conspicuous examples of this structure, and the last has long been recognized as such. The nature of the surrounding nebulosities has been inferred from the emission character of their spectra.

These results, it may be emphasized, were obtained with the

[1] Seed 30 plates were used in order to shorten the exposures and hence avoid changing conditions. It is probable that somewhat smaller images may occasionally be obtained by using plates having a finer grain. With longer exposures, the images of the condensations build up as those of ordinary stars even where both are superposed on an unresolved background. The sharper images of the short exposures, however, offer the more critical test. There are no indications whatsoever of penumbra appearing on long exposures, such as were described by Roberts and suggested by Ritchey.

highest resolving-power at present available. There is no longer any reason for retaining the term "nebulous stars" in referring to the condensations in Messier 33. They may be considered as actual stars and fit subjects for the ordinary methods of stellar research until evidence to the contrary is forthcoming.[1] This alone is an indication of immense distance, for no isolated system is known in which the brightest stars are dwarfs.

VARIABLE STARS IN MESSIER 33

In 1922 J. C. Duncan[2] announced the discovery of three variable stars within the area covered by Messier 33. His data were insufficient for determining the nature of the light-curves, and he refrained from suggesting any connection between the variables and the nebula. Shortly afterward, M. Wolf,[3] at Heidelberg, independently discovered the variability of the brightest of these stars.

The Mount Wilson collection now contains about eighty photographs of Messier 33. A survey of these with the blink-comparator has led to the discovery of forty-two additional variables, making a total of forty-five, besides two novae, within the area covered by the nebula. The similar concentrations of variables in Messier 31, the Magellanic Clouds, N.G.C. 6822, and in the globular star clusters, together with the scarcity of such faint variables in other regions of the sky,[4] especially in the high galactic latitudes of the spirals,

[1] It is still uncertain how far the resolution can be carried. The stars become more numerous with decreasing luminosity and, in the central region at least, appear to merge smoothly into the unresolved background. The luminosity function for the three brightest magnitudes agrees very closely with Kapteyn's luminosity function for stars in the vicinity of the sun, and the total luminosity of the nebulae is completely accounted for on the assumption that the similarity between the two functions extends throughout the entire range of magnitude. In Messier 31, however, the large central region gives no indication of resolution, although portions of the outer regions of the arms are as clearly resolved as those in Messier 33.

[2] *Publications of the Astronomical Society of the Pacific*, **34**, 290, 1922.

[3] *Astronomische Nachrichten*, **217**, 476, 1923. Duncan's paper was published in October, 1922. Wolf's paper was dated December 20, 1922, and published January 15, 1923. He does not mention Duncan's earlier announcement and hence, while there is no question of priority, Wolf's discovery may be accepted as independent.

[4] For ten fields well distributed in galactic latitude, other than those of the systems mentioned, there are in the Mount Wilson collection six or more plates showing stars fainter than 19.0 photographic magnitude. An examination of these with the blink-comparator has resulted in the detection of two doubtful cases of faint variables.

sufficiently establish the variables under discussion as connected with the nebula itself.

The stars are so closely crowded on photographs of the nebula that identification of the variables by the usual method of measures referred to the nucleus is unsatisfactory. They have, instead, been marked on the various plates and identified by numbers corresponding to those used in the text and tables.[1] Variables Nos. 1, 2, and 3 are those discovered by Duncan and bear the numbers assigned by him.

Series of comparison stars have been selected for each of the variables, and their photographic magnitudes have been derived from comparisons with Selected Area No. 45, which is conveniently situated some 30′ preceding the nucleus of the nebula. Twelve comparisons made with the 100-inch reflector were used in determining the scale and the zero-point. The exposures ranged from 20 to 40 minutes on Seed 30 plates, and were centered on various regions of the nebula in order to reduce uncertainties in the distance corrections. Two additional comparisons with Selected Area No. 21 furnished a satisfactory check.[2] The adopted magnitudes are based on those of the Selected Area down to about 18.5, but beyond this point they are extrapolations. The measures are very consistent, the average deviation of individual plates from the mean being less than 0.05 mag., and the greatest range for an individual star (on five plates), less than 0.4 mag. For stars in the central region fainter than about 19.0 the magnitudes are not reliable because of the background of unresolved nebulosity. In the other regions the measures are consistent to about 19.2, but beyond this they are, in general, unreliable.[3]

[1] The variables appear on the following plates: Plate XIV, all variables excepting Nos. 12, 13, 20, 21, 22, 37; Plate XV, variables Nos. 5, 9, 10, 11, 12, 13, 18, 20, 21, 22, 23, 24, 25, 37, 40; Plate XVI, variables Nos. 14, 29, 30, 31, 32, 33, 41, 42, 43, 44, 45; and Plate XVII, variables Nos. 11, 12, 17, 18, 20, 21, 23, 27, 29, 37.

[2] In addition to these long exposures, ten comparisons with Selected Area No. 45 were made with the 60-inch, using exposures ranging from two to fifteen minutes, for the purpose of deriving magnitudes for the stars brighter than the seventeenth magnitude. Mr. Seares kindly furnished unpublished magnitudes on the international scale for the stars in the Selected Areas.

[3] For a discussion of the uncertainties in extrapolated magnitudes based on measures with scale-plates of long exposures with the 100-inch reflector, see *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, 62, 409, 1926.

Magnitudes of variables on individual plates were estimated to the nearest tenth by reference to the sequences of comparison stars, the results being listed in Table I. A few of the stars, No. 21 in particular, are so involved with the unresolved nebulosity that the magnitudes have little or no meaning except at maximum. For most of the stars, however, it is believed that the data are sufficient to give fair values for the periods and the maximum magnitudes and some indications as to the general form of the light-curves. Magnitudes at minima, and hence the amplitudes of variation, are extremely uncertain. There are, in fact, several indications that the scale flattens out beyond about 19.0, and that the fainter magnitudes become increasingly too bright.

An examination of the data in Table I has established the nature of the variation of forty of the stars. Thirty-five are Cepheids, four are irregular, and one is very probably of an eclipsing type. Satisfactory solutions have not been found for the remaining five, but most of them are probably Cepheids.

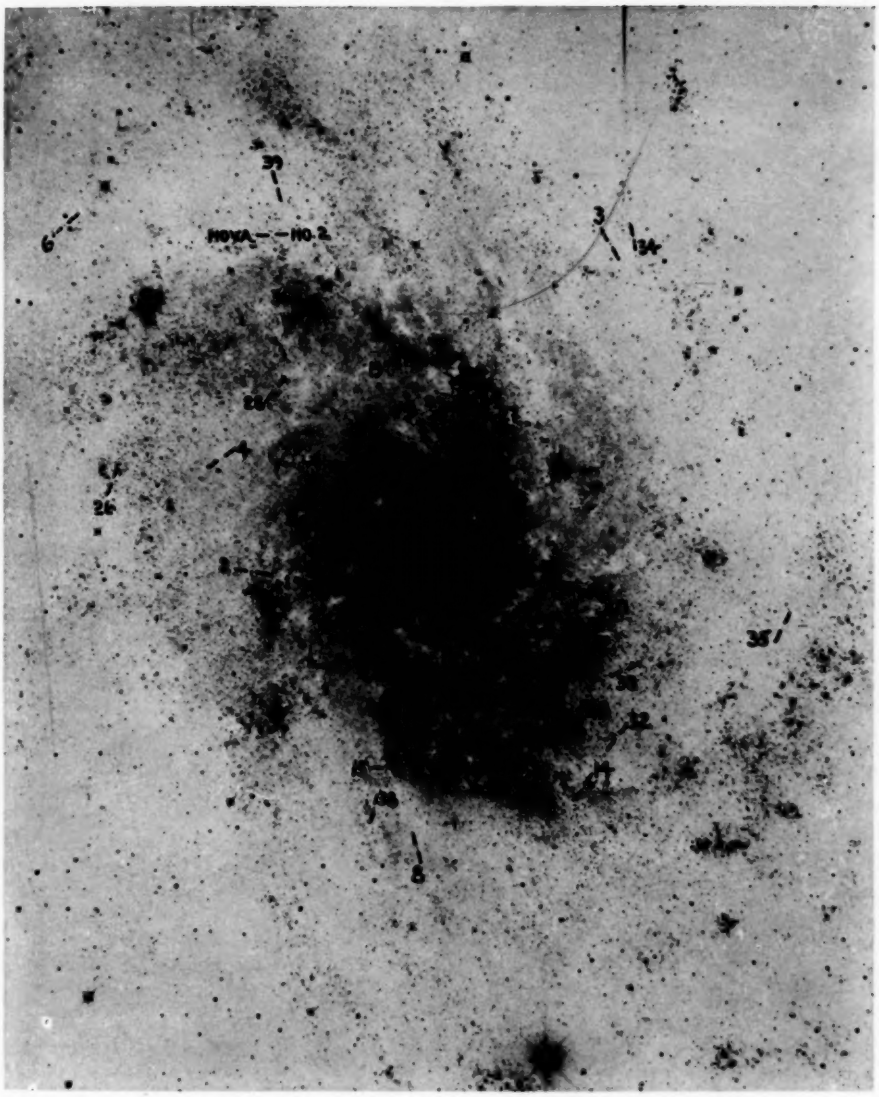
CEPHEID VARIABLES

The criteria used in determining Cepheid variation were: periods between two and one hundred days, amplitudes less than 2 mag., rapid brightening and slower fading, and no conspicuous secondary maxima. The limits for the first two of these are arbitrary, and, indeed, are more generous than necessary for the data under discussion. The maximum amplitude encountered is 1.2 mag., and the periods range from thirteen to seventy days.[1] The magnitudes at maximum range from 18.0 to 19.1. Minimum, and hence median, magnitudes are in general too uncertain for statistical discussion, but, although in several cases the minima are below the limits of the observations, there is no reason to suppose that any of the amplitudes approach the limiting value mentioned above.

In order to emphasize their Cepheid characteristics, several of the better-determined normal curves are illustrated in Figure 1.

[1] That some or all of the periods might be very close to one day, or some exact submultiple, was fully realized, and sufficient plates were obtained at different hour angles on the same night to eliminate this possibility. Each of the Cepheids was followed through several cycles after the period had been determined, and it is believed that the adopted elements are reliable.

PLATE XIV



Negative Ri 115. Exposure 8.5 hours, Seed 23 plate, 60-inch reflector, August 5, 6, 7, 1910. Scale 1 mm = 14″.2. Nova No. 2 (1925) and all variables excepting Nos. 12, 13, 20, 21, 22, and 37 are marked.

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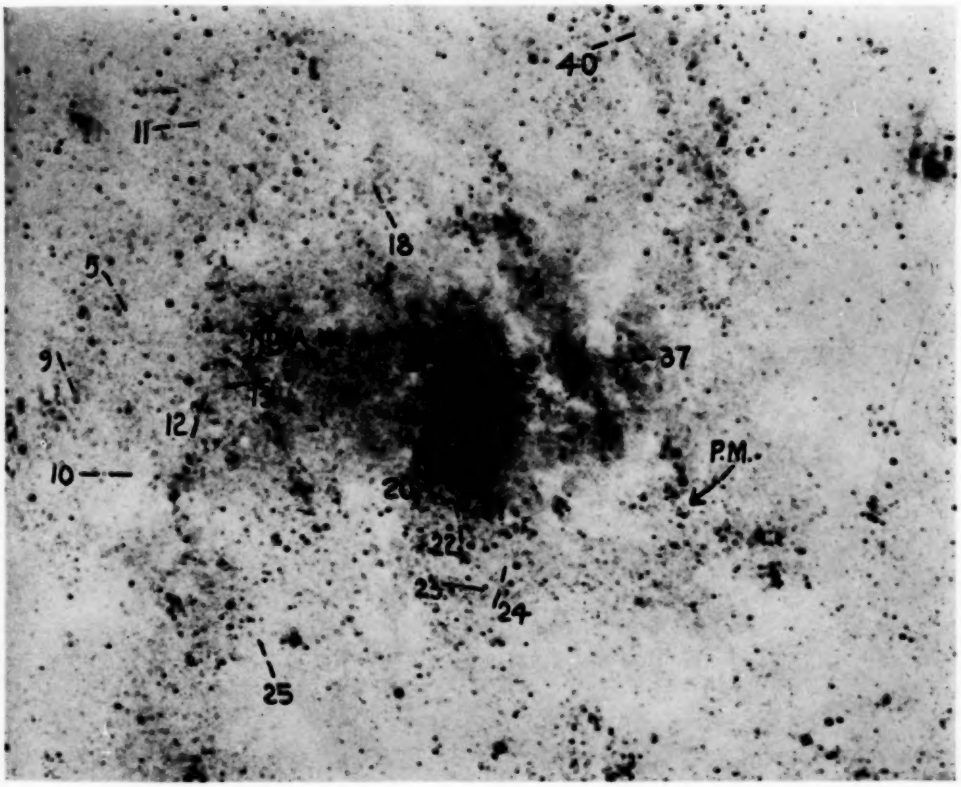
PLATE XV



Negative H 584. Exposure 90 minutes, Seed 30 plate, 100-inch reflector, July 26, 1925. Scale 1 mm=3″.8. Nova No. 1 (1919) and variables Nos. 5, 9, 10, 11, 12, 13, 18, 20, 21, 22, 23, 24, 25, 37, and 40 are marked. The star marked P.M., pg. mag. 18.25, is moving about 10″ per century in the direction of the arrow.

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PLATE XVI

Negative H 485. Exposure 30 minutes, Seed 30 plate, 100-inch reflector, November 26, 1925, centered about 7′ south preceding the nucleus of Messier 33. Scale, 1 mm = 4″.3. Variables Nos. 14, 29, 30, 31, 32, 33, 41, 42, 43, 44, and 45 are marked. The mean diameter of the smallest star images is approximately 0″.5.

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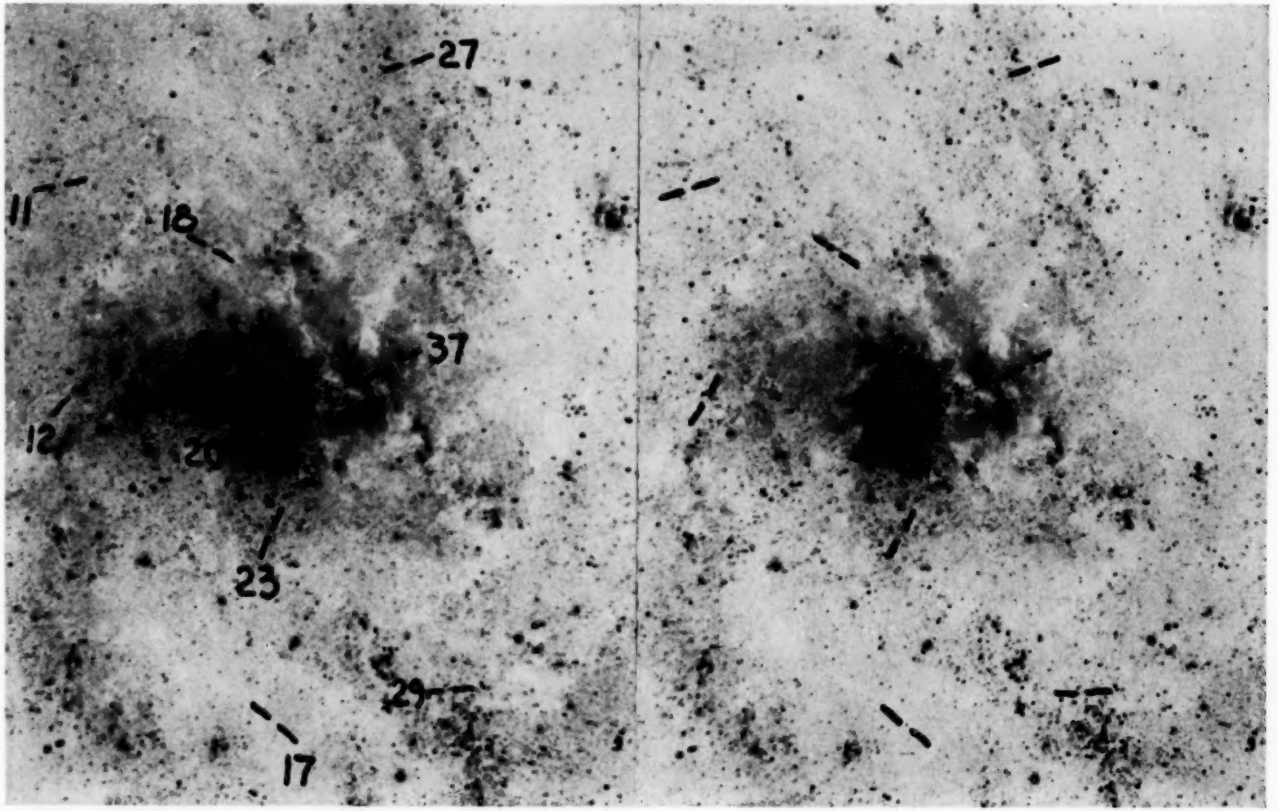
PLATE XVII



Light-Changes of Variables in Messier 33. Negatives H 446 (left) and H 475 (right), both made with the 100-inch reflector. Scale, 1 mm = 5".2. All of the variables except No. 21 are Cepheids.

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TABLE I

PHOTOGRAPHIC MAGNITUDES OF VARIABLES IN MESSIER 33

PLATE*	QUALITY	J.D.†	CEPHEIDS‡						
			3	4	5	7	9	10	11
R 15	F	18560.6	18.6	19.3	19.2	19.3	19.3	19.1	19.15
115	E	18890.6	18.3	19.3	19.4+	18.7	18.95	19.0	19.15
D 34	P	22197.6	19.0	19.3	19.0	19.2	19.0	18.9	18.8
P133	F	2307.4	18.45	19.3+	19.3+	19.2	18.8	19.15	19.2+
134	P	2311.4	18.45	19.0	19.3+	19.1	19.2	19.2	18.95
138	G	2314.4	18.7	18.1	18.8	18.75	19.3	19.2	18.75
139	P	2347.8	18.25	18.7	19.3	18.9	18.95	18.9	19.2
D 48	G	2550.6	19.0	19.15	19.3+	19.3	18.7	18.8	18.85
83	P	2611.0	19.0	19.2+	19.2+	18.6	.....	.....	.....
86	F	2612.5	18.95	19.3	19.3+	18.8	19.0	18.65	19.15
108	P	2669.4	.....	18.1	.....	.....	19.0	19.15	18.9+
119	F	2694.3	18.95	19.3+	18.85	.....	19.2+	18.5	18.9
130	G	2698.3	19.1	18.2	19.3	19.2	19.3	18.95	.....
198	G	2903.2	18.9	19.3	19.3+	18.85	19.3	18.9	18.8
205	G	2907.2	18.9	19.15	19.4+	18.85	19.3	18.95	19.0
A 4	F	2966.5	19.2	.....	.....	19.1+	18.75	18.65	18.95
8	G	3000.4	19.1	18.1	18.85	19.2	19.3	19.15	19.3
40	G	3319.6	19.0	19.15	19.3	19.3	19.3	18.7	18.8
B 65	G	3322.1	19.0	19.4	18.85	19.1	19.3	18.95	18.9
M335	G	3344.5	18.9	19.2+	19.3+	19.15	18.7	19.1	18.9
H417	E	4004.7	19.3	19.4	19.3+	19.3	18.9	18.5	19.1
D239	G	4024.6	18.8	19.3	19.1	19.15	18.65	19.0	18.9
245	G	4025.6	18.9	19.3	19.2	19.2	18.75	18.9	18.9
S354	F	4026.6	18.85	19.3	19.1+	19.3	18.9	18.95	18.95
357	F	4027.6	18.9	19.3	19.1+	19.2+	19.0+	18.95	18.9+
363	G	4030.6	19.1	19.2+	19.1+	19.3	19.2	19.1	19.0
H422	G	4031.6	19.1	19.4	19.3+	19.2	19.3	19.0	19.15
428	F	4033.6	19.0	19.4+	19.4+	19.2	19.3	19.0	19.15
S367	P	4049.6	19.1	19.0	.....	18.9	.....	.....	18.85
373	P	4050.6	19.2	19.2	19.1	19.1	.....	19.15	18.9
H433	F	4054.6	18.35	19.3+	18.9	19.1	19.1+	19.1	19.0
S375	F	4055.6	18.45	19.25	19.05	19.3	19.3	19.15	19.1
383	F	4056.5	18.4	19.3	19.1+	19.2	.....	19.1	19.0
H441	F	4058.5	18.45	19.25	19.1+	18.9+	.....	.....	.....
446	E	4059.5	18.45	19.15	19.25	19.2	19.0	19.15	19.15
452	P	4085.5	19.3	19.5	19.2+	19.2	18.95	18.7	19.2+
S399	P	4089.5	19.2	19.3	19.1+	19.3+	19.0+	18.85	19.1+
H458	F	4090.5	19.3	19.0	19.3+	19.3+	19.3	18.85	18.9
465	F	4092.5	19.2	18.4	19.1+	18.8+	19.3	18.95	18.85
467	F	4108.4	18.6	19.4	19.3	19.3	19.1	19.0	19.2
472	F	4109.4	18.65	19.5	19.2	19.2	19.4+	19.0	19.3+
S407	F	4113.4	18.7	19.3	19.1+	19.3	19.0+	19.15	19.15
H475	E	4115.4	18.75	19.35	19.3	19.2	19.3	19.15	19.0
486	E	4116.3	19.0	19.3	19.3	19.3	19.0	.....	18.75
503	F	4147.3	18.8	18.45	19.3+	18.6	19.2	18.6	19.1
509	G	4149.3	18.8	18.6	19.2+	18.7	19.2	18.6	19.0+
517	F	4171.3	19.1	19.15	19.2	19.2	19.3	18.95	19.0
577	G	4348.6	18.3	19.4	19.2	19.2	18.9	18.95	19.1
D253	F	4351.6	18.4	19.4	19.2	19.2	19.0	18.7	18.9
H584	E	4357.6	18.6	19.5	19.1	19.2	19.3	18.55	19.0
S497	P	4358.6	18.65	.....	18.9	.....	.....	.....	.....
H588	G	4377.6	19.3	19.1	19.1	19.15	19.3	18.95	18.8
503	G	4378.6	19.3	19.25	19.2	19.2	19.3	18.95	18.9
600	G	4379.6	.....	19.3+	.....	.....	.....	.....	18.9
D277	E	4384.6	19.2	19.4	19.4	18.9	18.7	19.0	19.25
288	E	4387.6	18.45	19.5	18.8	18.6	18.9	19.1	19.2
H606	P	4405.5	18.85	19.1	19.3	19.2	18.75	19.05	19.0
611	G	4414.5	19.1	19.35	19.25	18.6	19.4	18.85	19.2+

* The prefixed letters refer to the observers: R = Ritchey, D = Duncan, P = Pease, A and B = Humason, M = van Maanen, S and H = Hubble.

† Add 2,400,000 to the first three dates, and 2,420,000 to the others. The decimal indicates Pacific Mean Time. In order to conform to the change to Civil Day Time, 0.5 must be added to each date.

‡ A plus sign following a magnitude indicates that the variable was invisible, but certainly fainter than the value indicated.

TABLE I—*Continued*

Plate	Quality	J.D.	Cepheids						
			12	16	17	18	19	20	22
R 15	F	18560.6	19.0+	19.0	18.95	18.8	18.1	B	.....
115	E	18890.6	19.0	18.8	19.2	18.55	18.2	18.55	19.4
D 34	P	22197.6	.....	18.7	19.15	18.75	18.9	.....	.....
P133	F	2307.4	19.2+	19.2	19.4+	19.05	18.7	18.65	19.1
134	P	2311.4	19.0	19.2	18.75	19.05	18.7	18.75	19.3
138	G	2314.4	18.6	19.3+	18.95	19.05	18.7	18.9	19.2+
139	P	2347.8	19.2+	19.2	19.4+	19.05	18.7	18.75	.....
D 48	G	2550.6	.....	19.1	19.4	19.0	18.1	B	19.1+
83	P	2611.0	.....	.....	.....	.....	18.4	.....	.....
86	F	2612.5	19.2+	19.3	19.4	18.95	18.35	.....	.....
108	P	2669.4	.....	.....	19.2+	18.7	18.5	.....	.....
119	F	2694.3	19.2+	19.2+	19.2+	18.7	18.8	.....	18.85
130	G	2698.3	19.2+	19.3	19.4+	18.55	18.6	18.4	19.0
198	G	2903.2	18.9	19.3	19.4+	18.65	18.8	18.9	19.1+
205	G	2907.2	19.3+	19.3	19.4	18.8	18.7	18.85	19.3+
A 4	F	2966.5	18.8	18.75	19.1	18.7	18.85	18.9	19.2+
8	G	3000.4	19.4	18.8	19.4	18.7	18.5	19.0	19.0
40	G	3319.6	19.4	19.1	19.2	18.75	18.4	19.0	19.3+
B 65	G	3322.1	19.3	19.3	19.4	18.8	18.4	18.9	19.4
M335	G	3344.5	19.4	19.4	19.4	18.5	18.8	18.3	19.4
H417	E	4004.7	18.6	19.3	19.4+	18.9	18.7	19.0	18.9
D239	G	4024.6	18.6	19.2	19.4	18.5	18.0	18.2	18.95
245	G	4025.6	18.6	19.2	19.4+	18.55	18.2	18.3	18.9
S354	F	4026.6	.....	19.2+	19.4+	18.6	18.1	.....	.....
357	F	4027.6	18.75	19.0+	19.2+	18.7	18.3	.....	.....
363	G	4030.6	18.9	19.1+	19.4+	18.75	18.35	18.6	.....
H422	G	4031.6	19.1	19.2	19.4+	18.75	18.35	18.6	19.2
428	F	4033.6	19.1	18.6	19.4+	18.8	18.4	18.65	19.3
S367	P	4049.6	.....	.....	19.3+	.....	18.85	.....	.....
373	P	4050.6	.....	18.7	19.4+	.....	18.8	.....	.....
H433	F	4054.6	19.3+	19.05	19.4+	18.9	18.7	18.9+	19.3
S375	F	4055.6	.....	19.1	19.4+	18.75	18.8	18.9+	19.3
383	F	4056.5	.....	19.1	19.4+	18.5	18.7	.....	19.3
H441	F	4058.5	19.3+	.....	18.75	18.45	18.7	.....	.....
446	E	4059.5	19.4	19.2	18.75	18.55	18.75	18.6	19.35
452	P	4085.5	19.1+	18.8	19.15	19.0	18.25	19.0	19.3+
S399	P	4089.5	18.9	19.0+	19.1+	.....	18.35	.....	.....
H458	F	4090.5	18.6	19.1	19.3	19.55	18.3	19.1	18.9
465	F	4092.5	.....	19.2+	19.3+	18.5	18.45	19.1	19.05
467	F	4108.4	19.3+	19.05	19.0	19.0	18.7	18.65	19.2+
472	F	4109.4	19.3	19.2	19.15	18.9	18.7	18.85	19.4
S407	F	4113.4	.....	19.2+	19.4+	.....	18.8	.....	.....
H475	E	4115.4	18.95	19.5	19.4+	19.0	18.7	19.0	19.05
486	E	4116.3	19.1	19.4	19.4	19.05	18.7	19.0	19.1
503	F	4147.3	19.1+	19.2	.....	19.1	18.45	.....	.....
509	G	4149.3	19.4	19.2	19.4+	19.1	18.55	18.9	19.4
517	F	4171.3	19.4	19.2	.....	18.8	18.65	18.2	19.4
577	G	4348.6	19.3+	18.7	19.3+	19.1	18.0	18.1	19.4
D253	F	4351.6	18.8	18.85	19.4+	19.1	18.1	18.45	19.1
H584	E	4357.6	19.1	19.4	19.45	19.0	18.3	18.6	19.2
S497	P	4358.6	.....	.....	.....	.....	18.15	.....	.....
H588	G	4377.6	19.1	19.4	19.4+	18.85	18.7	19.0	19.2
593	G	4378.6	19.1	19.4	19.4+	18.9	18.8	19.0	19.2
600	G	4379.6	19.1+	19.4	.....	.....	18.8	19.0	.....
D277	E	4384.6	19.1+	18.75	18.8	18.9	18.7	18.3	19.5
288	E	4387.6	19.4	19.2	19.0	18.9	18.7	18.4	19.4
H606	P	4405.5	19.1+	19.1	19.4+	18.75	18.0	.....	.....
611	G	4414.5	18.8	19.4+	19.1	18.9	18.2	19.0	19.35

TABLE I—*Continued*

Plate	Quality	J.D.	Cepheids						
			23	24	25	26	27	28	29
R 15	F	18560.6	.....	.....	.....	19.2	19.05	19.6	18.8
115	E	18890.6	18.95	19.3	19.3	19.6	19.3	19.7	19.2
D 34	P	22197.6	.....	.....	.....	19.4+	19.2	19.5+	19.0
P133	F	2307.4	19.15	19.3	.....	19.4+	19.0	19.6+	19.25
134	P	2311.4	.....	.....	19.6+	19.5+	19.1	19.6+	19.2+
138	G	2314.4	19.3	18.8	19.6+	19.4+	19.2	18.85	19.3
139	P	2347.8	19.2+	19.4+	19.4	19.1	19.2	19.6+	19.3
D 48	G	2550.6	19.2+	19.4	19.3	19.4+	18.75	19.5+	18.5
83	P	2611.0	.....	.....	.....	.....	.....	18.8	19.2+
86	F	2612.5	19.2+	19.1	19.4	19.5+	19.3	19.1	19.3
108	P	2669.4	.....	.....	.....	.....	.....	.....	.....
119	F	2694.3	.....	.....	18.85	.....	.....	.....	19.2
130	G	2698.3	19.2+	19.4+	19.3	19.4	19.2	19.5+	19.1
198	G	2903.2	19.3	.....	19.5+	19.4	19.2	19.7	18.6
205	G	2907.2	19.0	19.2	19.5+	19.2	19.3	19.6+	19.3
A 4	F	2966.5	19.2+	.....	19.2	.....	19.2	.....	19.3
8	G	3000.4	18.85	19.3	19.7	19.35	18.7	19.6	19.1
40	G	3319.6	19.4+	19.1	.....	19.2+	19.2	19.1	19.1
B 65	G	3322.1	19.4	18.9	19.6+	19.2	19.3	19.6	19.1
M335	G	3344.5	19.3+	19.3+	19.5+	.....	19.1	.....	19.3
H417	E	4004.7	19.0	19.05	19.5+	19.5	19.4	19.5	18.7
D239	G	4024.6	19.3	19.4+	19.1	19.4	19.3	18.75	19.3
245	G	4025.6	19.4	19.4+	19.15	19.3	19.3	19.0	19.3
S354	F	4026.6	.....	.....	.....	19.5+	19.3	19.3	19.2+
357	F	4027.6	.....	.....	.....	.....	.....	19.5	.....
363	G	4030.6	.....	.....	.....	19.4+	19.2	19.5+	19.2+
H422	G	4031.6	18.8	19.1	19.5+	.....	19.2	.....	19.3
428	F	4033.6	19.1	19.2	19.5+	19.4+	18.7	19.5+	19.3
S367	P	4049.6	.....	.....	19.3	19.4+	.....	19.5+	.....
373	P	4050.6	.....	.....	19.1	19.4+	.....	19.6+	.....
H433	F	4054.6	.....	.....	.....	.....	18.75	.....	19.2
S375	F	4055.6	.....	.....	19.45	19.6+	18.7	19.7	19.2
383	F	4056.5	.....	.....	.....	19.4+	18.75	19.6+	19.3
H441	F	4058.5	.....	.....	.....	.....	.....	.....	.....
446	E	4059.5	19.0	19.35	19.6	19.4+	19.1	19.5+	19.3
452	P	4085.5	18.85	19.4	19.6	19.4+	19.3+	19.5+	19.1
S399	P	4089.5	.....	.....	.....	19.1	19.2+	19.6+	19.2
H458	F	4090.5	19.3	19.4+	19.4	19.0	19.2	19.6+	19.2+
465	F	4092.5	19.2+	19.2	18.9	19.1	19.3	.....	19.2+
467	F	4108.4	19.4	19.2	19.4+	19.4+	19.0	19.5+	19.3
472	F	4109.4	19.4	.....	.....	19.4+	19.25	19.5+	19.2
S407	F	4113.4	18.7	.....	.....	19.2	.....	19.5+	18.5
H475	E	4115.4	19.2	19.5	19.7	19.2	19.3	19.6	18.7
486	E	4116.3	19.25	19.4+	19.4	19.1	19.2+	18.9	18.75
503	F	4147.3	19.4	19.2	19.4	19.4+	18.9	19.5+	18.65
509	G	4149.3	19.4	19.2	19.4	.....	19.0	.....	18.6
517	F	4171.3	19.4+	18.85	18.85	19.4+	18.95	19.5+	19.3
577	G	4348.6	19.4+	19.4+	19.35	19.2	18.9	19.5+	19.2
D253	F	4351.6	19.2+	18.8	19.4+	19.4+	19.1	19.5+	19.3
H584	E	4357.6	18.8	19.4	19.7	19.6	19.25	19.1	19.3
S497	P	4358.6	.....	.....	.....	19.4+	.....	18.9	.....
H588	G	4377.6	19.4	18.95	19.5+	19.4+	19.2	19.0	19.0
593	G	4378.6	19.4	18.95	19.5+	19.4+	.....	19.0	19.0
600	G	4379.6	19.3+	.....	.....	19.4+	19.0+	19.3	19.1
D277	E	4384.6	18.95	19.5+	19.6	19.6+	19.3	19.7+	19.2
288	E	4387.6	19.25	19.5+	18.9	19.6+	19.2	19.7	19.35
H606	P	4405.5	19.0+	18.9	.....	19.4+	.....	19.5+	18.65
611	G	4414.5	19.0+	19.2+	18.9	19.1	18.7	18.85	19.2

TABLE I—*Continued*

Plate	Quality	J.D.	Cepheids						
			30	31	33	34	35	36	37
R 15	F	18560.6	18.9	18.3	.....	19.4	19.4	18.85	.....
115	E	18890.6	19.1	19.3	19.7+	19.1	19.2	19.1	19.4+
D 34	P	22197.6	18.7	19.3	.....	19.2	.....	19.0	.....
P133	F	2307.4	19.15	19.0	19.3	19.2	19.2	18.8	.....
134	P	2311.4	.....	19.1	19.7	19.3	19.3	18.45	19.4+
138	G	2314.4	19.2	19.15	19.5+	19.4	19.3	18.65	19.5+
139	P	2347.8	19.15	19.15	19.3+	19.1	19.4	18.65	.....
D 48	G	2550.6	19.1	19.15	18.8	19.0	19.3	19.1	.....
83	P	2611.0	18.6	.....	.....	18.9	19.4	19.0	.....
86	F	2612.5	18.9	19.15	.....	18.95	19.45	19.0	.....
108	P	2669.4	.....	.....	.....	.....	.....	.....	.....
119	F	2694.3	18.3	19.15	.....	18.75	18.7	19.0	19.2+
130	G	2698.3	18.45	.....	19.5	18.85	18.9	19.0	.....
198	G	2903.2	19.15	19.0	19.3+	19.3	18.65	19.0	19.3+
205	G	2907.2	19.15	19.0	19.7	19.3	18.7	19.0	19.5+
A 4	F	2966.5	18.65	18.2	19.4	.....	18.7	19.0	19.05
8	G	3000.4	19.15	18.35	18.75	19.4	18.9	18.75	19.4+
40	G	3319.6	19.15	19.0	19.7+	18.85	19.2+	18.7	18.75
B 65	G	3322.1	19.15	19.3	19.7+	19.2	19.4	18.8	19.0
M335	G	3344.5	18.5	18.5	19.7+	19.1	19.2+	19.1	19.4
H417	E	4004.7	19.1	19.2	18.75	18.75	18.8	18.8	19.1
D239	G	4024.6	18.8	19.0	19.5	18.55	19.4+	18.9	19.0
245	G	4025.6	18.45	19.15	18.75	18.6	19.4+	18.8	19.0
S354	F	4026.6	18.3	19.15	.....	18.6	19.4	18.8	.....
357	F	4027.6	18.3	19.15	.....	.....	.....	18.7	.....
363	G	4030.6	18.5	19.2	19.1+	19.2	18.8	18.55	19.3
H422	G	4031.6	18.4	19.2	19.2	19.1+	18.65	18.5	19.5
428	F	4033.6	18.45	19.1	19.5+	19.0	18.65	18.5	19.4
S367	P	4049.6	19.1	18.3	.....	18.6	.....	19.0	19.1+
373	P	4050.6	.....	18.4	.....	18.6	19.2+	19.0	.....
H433	F	4054.6	19.1	18.4	.....	.....	.....	19.0	19.3+
S375	F	4055.6	19.1	18.45	19.7+	19.1	19.5	19.1	19.1+
383	F	4056.5	19.2	18.55	19.3+	19.2	19.2+	19.0	19.3+
H441	F	4058.5	.....	.....	.....	.....	.....	.....	18.95
446	E	4059.5	19.15	18.95	19.7+	19.5	19.2	19.1	18.85
452	P	4085.5	18.9	18.2	19.6+	19.4	19.4+	19.0	19.3+
S399	P	4089.5	19.0	18.4	.....	19.1	19.2	19.0	19.2+
H458	F	4090.5	19.1	18.45	18.9	18.75	19.1	19.0	19.3+
465	F	4092.5	19.1	18.8	19.4	18.6	18.6	.....	19.35
467	F	4108.4	19.2	19.1	18.75	19.3	19.3	18.65	19.3
472	F	4109.4	19.25	19.2	18.9	19.4	19.4+	18.7	19.4+
S407	F	4113.4	19.15	19.2	19.1+	18.55	19.45	19.0	.....
H475	E	4115.4	18.95	19.2	19.7	18.6	19.45	19.0	19.0
486	E	4116.3	18.8	19.3	19.5	18.75	19.45	19.0	19.0
503	F	4147.3	19.2	19.3	19.5	19.2	.....	18.8	18.85
509	G	4149.3	19.2	19.15	19.0	19.2	.....	18.8	19.0
517	F	4171.3	18.5	18.85	19.1	19.2	19.4	18.5	19.3+
577	G	4348.6	18.3	18.3	19.3+	19.3	19.2	18.7	19.2+
D253	F	4351.6	18.45	18.5	19.3	19.3	19.2	18.7	19.3+
H584	E	4357.6	18.5	18.85	19.1	18.45	19.2	18.6	18.9
S497	P	4358.6	.....	.....	.....	18.5	.....	18.6	.....
H588	G	4377.6	19.2	19.2	19.0	19.2	19.0	19.0	18.9
593	G	4378.6	19.25	19.15	19.1	18.7	.....	19.0	18.9
600	G	4379.6	19.2	.....	19.2	.....	.....	19.1	19.2
D277	E	4384.6	19.25	18.2	19.7	18.9	19.4	18.6	19.3
288	E	4387.6	19.25	18.5	19.8+	19.15	19.3	18.45	19.5
H606	P	4405.5	18.9	19.1	19.3+	18.7	18.9	19.0	.....
611	G	4414.5	19.0	19.3	19.0	19.2	19.4	19.1	19.05

TABLE I—*Continued*

Plate	Quality	J.D.	Cepheids						
			38	39	40	41	42	43	44
R 15	F	18560.6	18.7	19.2	19.1+	19.1	18.45	18.8	.....
115	E	18890.6	19.1	19.5	19.2	19.25	19.0	19.3	19.2
D 34	P	22107.6	18.8	19.1+	19.1	.....	.....	.....	.....
P133	F	2307.4	19.1	19.1	19.4+	19.2	18.9	19.6	19.4
134	P	2311.4	18.2	19.3	19.0	18.7	19.0	18.7	19.2
138	G	2314.4	18.5	19.2+	19.3	19.1	19.0	18.95	18.6
139	P	2347.8	19.1	19.2+	19.4+	18.7	19.0	19.4+	19.1
D 48	G	2550.6	19.1	19.1	19.3+	19.25	19.0	19.6	19.4
83	P	2611.0	18.6	18.9	.....	.....	.....	.....	.....
86	F	2612.5	18.3	18.8	19.35	19.3	18.9	19.6	.....
108	P	2669.4	.....	.....	.....	.....	.....	.....	.....
119	F	2694.3	18.6	.....	19.4+	19.2	18.8	19.15	.....
130	G	2698.3	19.1	19.0+	19.3+	19.25	18.8	18.7	19.4
198	G	2903.2	18.95	18.8	19.4	19.1	18.6	19.4	19.3
205	G	2907.2	19.1	19.3	18.9	18.75	18.7	19.7	19.35
A 4	F	2966.5	18.6	.....	19.4+	19.25	18.8	19.4	19.2
8	G	3000.4	19.1	18.9	19.45	19.1	18.9	18.8	19.4
40	G	3319.6	18.6	19.2+	18.9	18.9	19.0	18.7	19.3
B 65	G	3322.1	18.8	19.2	19.3	19.1	18.9	18.8	19.4
M335	G	3344.5	18.6	.....	19.4	19.25	19.0	19.1	.....
H417	E	4004.7	19.0	19.1	18.95	18.95	18.9	18.95	18.8
D239	G	4024.6	19.0	19.2+	19.35	19.2	18.6	19.1	19.4
245	G	4025.6	18.9	19.2+	19.35	19.25	18.7	18.7	19.4
S354	F	4026.6	19.0	19.2+	.....	.....	18.7	18.7	19.3
357	F	4027.6	19.0	19.0+	.....	19.2	18.8	18.65	19.2
363	G	4030.6	19.0	19.2+	18.95	19.2	18.8	19.1	18.8
H422	G	4031.6	19.1	.....	19.15	19.25	18.9	19.3	18.6
428	F	4033.6	19.1	18.8	19.35	19.3	18.8	19.3	18.6
S367	P	4049.6	19.0	18.9	19.1+	19.1+	18.8+	19.0	19.3+
373	P	4050.6	18.8	19.0	19.1+	19.1	18.6	19.1	19.3
H433	F	4054.6	19.1	.....	19.3	18.6	18.45	19.4	19.3
S375	F	4055.6	19.1	19.2+	19.2	18.75	18.6	19.7	19.4
383	F	4056.6	.....	19.1+	18.9	18.75	18.45	19.3	19.4
H441	F	4058.5	.....	.....	19.1+	19.1	18.7	19.4	19.3
446	E	4059.5	19.1	19.0+	19.25	19.1	18.9	19.7	18.65
452	P	4085.5	19.15	19.1	19.3	19.25	18.6	19.3	19.3
S399	P	4089.5	18.2	19.2+	.....	19.2	.....	19.1	19.3
H458	F	4090.5	18.3	19.1+	19.35	18.6	18.8	18.8	18.8
465	F	4092.5	18.5	.....	19.4	18.8	18.8	18.95	18.6
467	F	4108.4	19.1	19.1+	18.95	18.7	18.9	18.7	19.4
472	F	4109.4	19.1	19.2+	19.15	18.7	18.8	18.9	19.4
S407	F	4113.4	18.4	19.1+	19.2+	18.95	18.45	.....	.....
H475	E	4115.4	18.35	18.8	19.35	19.1	18.6	19.2	19.5
486	E	4116.3	18.45	.....	19.3+	19.1	18.45	19.4	19.4
503	F	4147.3	18.8	18.75	19.0	18.9	18.6	18.6	.....
509	G	4149.3	19.1	.....	19.25	19.2	18.7	18.6	19.1
517	F	4171.3	18.7	19.1+	19.25	19.25	18.85	18.7	19.4
577	G	4348.6	18.8	19.3+	19.35	19.1	19.0	18.7	19.3
D253	F	4351.6	19.1	19.2+	19.15	19.2	18.85	19.1	19.6
H584	E	4357.6	19.1	18.8	19.35	19.2	18.55	19.35	19.6
S497	P	4358.6	.....	18.75	.....	.....	.....	.....	.....
H588	G	4377.6	19.1	19.1	19.25	18.65	19.0	19.4	19.4
593	G	4378.6	19.1	19.2+	18.95	18.65	19.0	19.7	19.4
600	G	4379.6	19.1	19.1+	19.0	18.7	19.0+	19.7	19.3
D277	E	4384.6	19.3	19.3+	19.45	19.2	18.85	19.5	19.3
288	E	4387.6	18.8	19.2+	19.5	19.3	18.6	19.1	19.6
H606	P	4405.5	19.1	18.8	19.1	19.1	19.0	19.7	19.2
611	G	4414.5	18.25	19.3	19.5+	18.7	18.85	19.2	19.6

TABLE I—*Continued*

Plate	Quality	J.D.	Irregular				Algol
			1	2	6*	13	8
R 15	F	18560.6	17.7	17.3	18.4	18.4	18.6
115	E	18890.6	17.2	17.4	18.6	18.25	18.6
D 34	P	22107.6	18.6	16.4	18.5	18.15	18.8
P133	F	2307.4	18.6	16.4	18.6	18.15	18.6
134	P	2311.4	18.55	16.3	18.6	18.05	18.8
138	G	2314.4	18.6	16.4	.....	17.95	18.7
139	P	2347.8	18.4	16.3	18.7	18.15	18.7
D 48	G	2550.6	18.6	16.5	18.6	18.1	18.7
83	P	2611.0	18.5	16.5	.....	18.2	18.7
86	F	2612.5	18.45	16.5	18.7	.....	18.7
108	P	2669.4	.....	16.6	.....	.....	.....
119	F	2694.3	.....	16.5	.....	18.05	18.7
130	G	2698.3	18.4	16.3	.....	18.1	18.8
198	G	2903.2	18.55	16.1	18.8	18.0	18.7
205	G	2907.2	18.45	16.0	18.7	18.1	18.7
A 4	F	2966.5	18.7	16.1	.....	18.05	18.7
8	G	3000.4	18.7	15.8	18.8	17.9	18.7
40	G	3319.6	18.5	16.3	.....	17.8	18.7
B 65	G	3322.1	18.5	16.1	18.8	17.9	18.8
M335	G	3344.5	18.4	16.3	.....	17.8	18.7
H417	E	4004.7	18.55	15.5	18.8	17.75	18.6
D239	G	4024.6	18.5	15.6	.....	17.9	18.75
245	G	4025.6	18.4	15.6	18.8	18.0	18.8
S354	F	4026.6	18.4	15.5	18.8	18.1	18.7
357	F	4027.6	18.5	15.5	.....	18.05	18.7
363	G	4030.6	18.4	15.5	18.8	18.15	18.7
H422	G	4031.6	18.6	15.5	.....	17.75	18.8
428	F	4033.6	18.55	15.6	18.8	17.7	18.75
S367	P	4049.6	18.4	15.5	.....	18.1	18.75
373	P	4050.6	18.3	15.6	18.8	18.1	18.75
H433	F	4054.6	18.4	15.6	.....	18.0	18.7
S375	F	4055.6	18.4	15.6	18.8	18.1	18.7
383	F	4056.5	18.4	15.5	.....	17.9	19.15
H441	F	4058.5	18.4	15.5	.....	17.75	18.6
446	E	4059.5	18.45	15.5	18.8	17.75	18.75
452	P	4085.5	18.3	15.5	18.8	18.1	18.7
S399	P	4089.5	18.3	15.6	18.8	18.15	18.75
H458	F	4090.5	18.5	15.5	.....	17.95	18.6
465	F	4092.5	18.3	15.6	.....	17.9	18.6
467	F	4108.4	18.3	15.5	18.8	17.9	18.7
472	F	4109.4	18.4	.....	18.8	17.95	18.7
S407	F	4113.4	18.35	15.6	18.8	18.0	18.6
H475	E	4115.4	18.5	15.5	18.8	17.8	18.75
486	E	4116.3	18.5	15.5	.....	17.75	18.7
503	F	4147.3	18.6	15.6	18.8	17.8	18.5
509	G	4149.3	18.5	15.5	18.8	17.9	18.6
517	F	4171.3	18.4	15.5	18.8	17.75	18.75
577	G	4348.6	18.4	15.7	18.8	17.9	18.7
D253	F	4351.6	18.45	15.7	18.8	17.85	18.7
H584	E	4357.6	18.3	15.9	18.8	17.8	18.6
S497	P	4358.6	18.5	15.6	.....	17.8	18.7
H588	G	4377.6	18.3	15.9	18.8	17.8	19.1
593	G	4378.6	18.4	15.9	18.8	17.75	18.6
600	G	4379.6	18.35	15.8	18.8	17.8	18.7
D277	E	4384.6	18.3	15.9	18.8	17.8	18.7
288	E	4387.6	18.4	16.0	18.8	17.85	19.2
H606	P	4405.5	18.3	15.9	18.8	17.8	18.7
611	G	4414.5	18.3	15.9	18.8	17.9	18.7

* Var. No. 6. Magnitudes are estimated.

TABLE I—*Continued*

Plate	Quality	J.D.	Undetermined				
			14	15	21	32	45
R 15	F	18560.6	.....	19.3+	.....	.....	18.8
115	E	18890.6	19.2	19.2	18.75	18.8	18.55
D 34	P	22197.6	.....	19.25	.....	.....	.....
P133	F	2307.4	19.35	19.35	18.5	19.1	19.0
134	P	2311.4	18.8	19.45	18.5	18.6	18.9
138	G	2314.4	18.9	19.35	18.65	19.35	18.9
139	P	2347.8	19.3+	19.25	18.85	19.2+	18.45
D 48	G	2550.6	19.2	19.35	18.85	18.85	19.0
83	P	2611.0	.....	.....	.....	.....	.....
86	F	2612.5	19.35	19.5	18.9	19.1	18.8
108	P	2669.4	.....	.....	.....	.....	.....
119	F	2694.3	.....	19.25	18.85	19.2	.....
130	G	2698.3	19.1	19.45	18.85	19.5+	19.0
198	G	2903.2	19.4	19.1	18.7	19.5+	19.0
205	G	2907.3	19.4	19.45	18.55	19.25	18.95
A 4	F	2966.5	19.05	.....	18.8	19.2+	18.7
8	G	3000.4	19.4	19.0	18.7	19.5	19.0
40	G	3319.6	19.2	19.3	18.45	19.5	18.4
B 65	G	3322.1	19.4	19.45	18.65	19.5	18.5
M335	G	3344.5	19.4	19.3+	18.9	19.4+	18.55
H417	E	4004.7	19.6	19.45	18.75	19.5+	18.45
D239	G	4024.6	19.05	19.4+	18.9	19.5+	18.95
245	G	4025.6	19.0	19.5	18.85	19.25	18.9
S354	F	4026.6	19.0	19.4+	.....	.....	.....
357	F	4027.6	.....	.....	.....	.....	.....
363	G	4030.6	19.0	.....	.....	.....	19.0
H422	G	4031.6	19.2	19.25	18.85	19.4	18.9
428	F	4033.6	19.2	19.45	18.85	19.0	19.0
S367	P	4049.6	19.2	.....	18.5	.....	.....
373	P	4050.6	.....	.....	18.5	.....	18.95
H433	F	4054.6	.....	19.4	18.55	.....	18.75
S375	F	4055.6	19.6+	19.1	18.55	19.2	18.9
383	F	4056.5	19.4+	19.1	18.55	19.2	18.9
H441	F	4058.5	.....	.....	18.55	.....	18.35
446	E	4059.5	19.2	19.4	18.55	19.5+	18.5
452	P	4085.5	19.2	19.45	18.75	19.4	18.55
S399	P	4089.5	.....	19.4+	.....	.....	.....
H458	F	4090.5	19.3	19.3+	18.75	18.9	18.7
465	F	4092.5	19.05	19.4+	18.75	19.2	18.6
467	F	4108.4	19.4	19.15	18.75	18.8	18.95
472	F	4109.4	.....	19.3	18.75	18.75	18.95
S407	F	4113.4	.....	19.3+	.....	.....	19.0
H475	E	4115.4	19.2	19.3	18.7	19.4	19.0
486	E	4116.3	19.2	.....	18.5	19.5	19.0
503	F	4147.3	19.5	19.4	18.7	18.6	18.5
509	G	4149.3	.....	19.3	18.75	18.8	18.3
517	F	4171.3	19.2	19.4	18.85	19.0	18.9
577	G	4348.6	18.8	19.4	18.8	18.6	18.8
D253	F	4351.6	18.95	19.2	18.8	19.4	18.7
H584	E	4357.6	19.05	19.1	18.75	19.4	18.25
S497	P	4358.6	18.9	.....	.....	.....	18.4
H588	G	4377.6	19.6	19.45	18.8	19.5	18.4
593	G	4378.6	19.4	19.5	18.85	19.5	18.45
600	G	4379.6	19.4+	19.3	18.7	18.8	18.5
D277	E	4384.6	19.3	19.5	18.65	19.45	18.5
288	E	4387.6	19.5	19.5	18.85	18.7	18.5
H606	P	4405.5	18.9	19.2	18.9	19.2	19.0
611	G	4414.5	19.1	19.3	19.0	19.4	19.1

The necessary data for the construction of light-curves for all the Cepheids are given in Table II. The period-luminosity relation is conspicuous among these Cepheids, and is illustrated in Figure 2, where logarithms of the periods are plotted against magnitudes at maximum. Long exposures on Cramer Iso plates through a visual filter, while not accurately calibrated as yet, indicate moderately large color-indices, quite consistent with solar-type spectra, for those Cepheids which could be identified.[1]

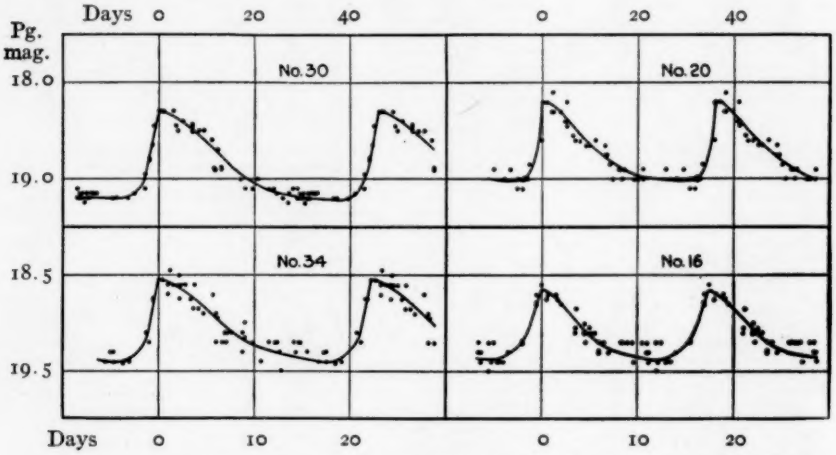

FIG. 1.—Mean light-curves for four Cepheids in Messier 33. The magnitudes are photographic.

These considerations all indicate that the variables are ordinary Cepheids, such as are known in the galactic system, the Magellanic Clouds, and the globular clusters, and hence that they can be employed for determining the distance of the spiral. An examination of Figure 2 suggests that the data are reasonably complete for maximum magnitudes down to about 18.8, but that for fainter maxima they are occasional and probably not representative.[2] This restriction, together with the limited range and considerable scattering, de-

[1] This conclusion is based on the fact that for stars having the same order of photographic magnitude the Cepheids fall about midway between the brightest and faintest photovisual images.

[2] Eighteen of the Cepheids are within 5′ of the nucleus, and seventeen are more distant. The two groups are thoroughly intermingled on the plot in Fig. 2, and show no appreciable difference that can be attributed to the influence of unresolved nebulosity.

TABLE II

CEPHEIDS IN MESSIER 33

VAR. NO.	PERIOD		PG. MAG.		EPOCH J.D.	DISTANCE FROM NUCLEUS	POSITION ANGLE‡	IDENTIFICATION PLATES
	Days	Log *P*	Maximum	Range*				
10†...	69.50	1.842	18.55	(0.6)	2424004.5	188″	105°	XIV, XV
19....	54.706	1.738	18.0	.8	4018.0	346	15	XIV
30....	46.03	1.663	18.3	.9	4026.0	301	250	XIV, XVI
3....	41.68	1.620	18.35	0.9	4054.5	633	330	XIV
31....	37.33	1.572	18.25	1.0	4084.0	337	245	XIV, XVI
29....	36.31	1.560	18.55	(0.8)	4111.5	237	210	XIV, XVI, XVII
20....	35.95	1.556	18.2	(.8)	4024.0	53	160	XV, XVII
36....	35.80	1.554	18.5	(.5)	4028.0	506	35	XIV
18....	34.00	1.531	18.5	(.6)	4057.0	104	30	XIV, XVII
35....	30.514	1.484	18.65	(.8)	4031.0	665	255	XIV
42....	30.34	1.482	18.45	(.5)	4051.0	323	225	XIV, XVI
44....	30.123	1.479	18.6	0.8	4029.5	353	230	XIV, XVI
4....	27.366	1.437	18.25	1.2	4091.5	444	70	XIV
7....	26.556	1.424	18.6	(0.7)	4147.0	296	60	XIV
38....	25.04	1.398	18.2	.9	4089.0	523	165	XIV
11....	23.43	1.370	18.8	(.5)	4116.0	201	50	XIV, XV, XVII
17....	23.297	1.367	18.8	(.9)	4058.5	229	180	XIV, XVII
26....	23.258	1.366	19.1	(.7)	4089.5	618	80	XIV
27....	22.448	1.351	18.7	(.6)	4054.5	233	340	XIV, XVII
34....	22.16	1.346	18.55	(.9)	4024.5	668	325	XIV
22....	21.816	1.339	18.9	(.5)	4002.5	64	175	XV
12....	21.681	1.336	18.6	(0.9)	4024.5	140	95	XV, XVII
33....	20.50	1.312	18.75	(1.1)	4004.5	430	235	XIV, XVI
43....	20.166	1.305	18.7	1.0	4025.5	424	215	XIV, XVI
9....	18.90	1.277	18.7	(0.6)	4023.0	204	90	XIV, XV
28....	18.58	1.269	18.85	(.8)	4116.5	412	50	XIV
41....	17.98	1.255	18.65	(.6)	4090.5	321	235	XIV, XVI
37....	17.602	1.246	18.9	(.8)	4058.0	77	280	XV, XVII
16....	17.50	1.243	18.7	(.7)	4033.0	239	170	XIV
39....	16.166	1.208	18.8	(.7)	4114.0	722	25	XIV
5....	14.60	1.165	18.85	(.6)	4051.5	186	75	XIV, XV
23....	13.564	1.132	18.85	(.6)	4112.5	96	180	XIV, XV, XVII
25....	13.438	1.129	18.95	(.8)	4386.0	169	140	XIV, XV
40†...	12.922	1.111	18.95	(.5)	4003.5	188	340	XIV, XV
24....	12.894	1.110	18.85	(0.7)	3990.0	96	190	XIV, XV

* The amplitudes, in general, are very uncertain; the few which seem to be fairly reliable are printed without parentheses.

† In the preliminary notice (*Publications of the American Astronomical Society*, 4, 261, 1925) the periods of Nos. 10 and 40 were given as 19.6 and 26.0 days, respectively. New observational data have led to the revised values listed above. The period previously published for No. 15 was also found to be erroneous.

‡ Position angles are measured from the north through the east (following side). The accuracy of both position angles and distances is sufficient only to indicate the regions in which the variables are located. Identification may be made on the various plates.

tracts materially from the significance of correlation curves computed from the data, and recourse was had to a direct cut-and-try method for determining $m-M$, the modulus of the distance.

Shapley's period-luminosity curve,[1] expressing the relation between log P and median photographic magnitude for more than a hundred Cepheids in a single system, the small Magellanic Cloud, is the latest and most reliable standard for comparison. The individual points were replotted, using magnitudes at maxima instead of median magnitudes,[2] and a corresponding diagram on transparent paper was constructed, to the same scale, for the Cepheids in Messier 33. The latter was laid upon the former so that the ordinates for log P were superposed and then adjusted to the best fit by shifting along the magnitude axis. This method assumes that the amplitudes of the Cepheids in the two systems are similar; otherwise, an error amounting to half the systematic difference in the amplitudes is introduced. In general, the two diagrams are comparable, although the scattering among the Cepheids in the spiral appears to be slightly less than among those in the cloud, while the deficiencies in the spiral for the fainter magnitudes and shorter periods are strikingly evident.

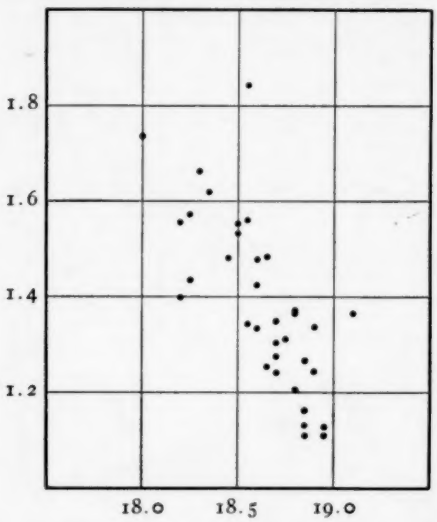

FIG. 2.—Period-luminosity relation among Cepheids in Messier 33. Photographic magnitudes at maximum (abscissae) are plotted against logarithms of period in days.

The best fit, illustrated in Figure 3, appears to be for a difference of 4.55 mag. in the moduli of the two systems, and the adjustment is sensibly unbalanced by a shift of 0.1 mag. in either direction. The

[1] *Harvard Circular*, No. 280, 1925.

[2] In two cases, where the amplitudes were not specifically stated, probable values were derived from a curve expressing the relation between amplitude and log P.

latest value[1] for the modulus of the cloud is 17.55; hence for Messier 33

$$m-M=22.1$$
$$\pi=0''.000,0038$$
$$\text{Distance}=263{,}000 \text{ parsecs}$$
$$=850{,}000 \text{ light-years.}^{2}$$

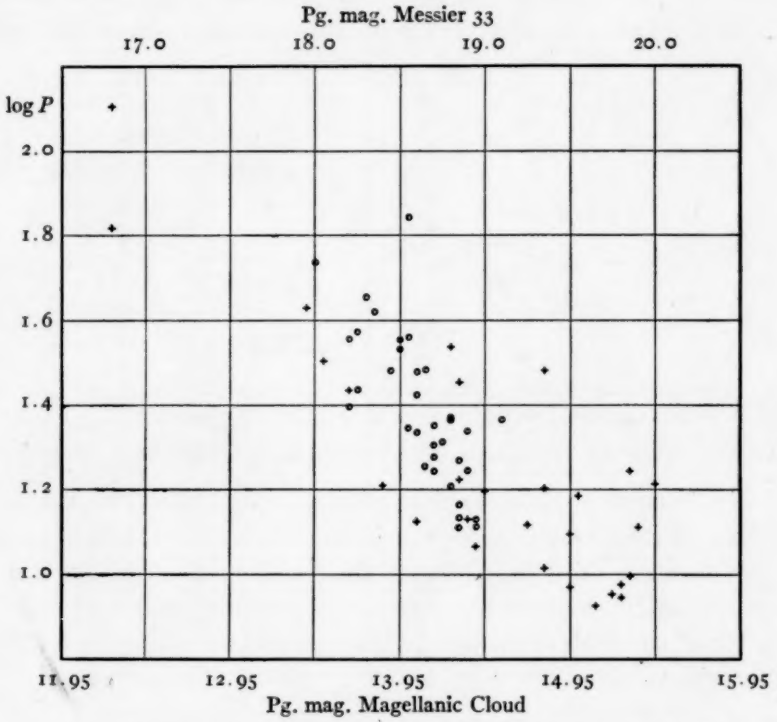

FIG. 3.—Comparison of period-luminosity relations for Cepheids in Messier 33 (circles) and in the Small Magellanic Cloud (crosses).

The accuracy of the results, the universal validity of the Cepheid criterion being assumed, depends largely upon the accuracy of the

[1] *Harvard Circular*, No. 280, 1925.

[2] The values published in the preliminary notice, derived from twenty-two Cepheids and Shapley's unrevised period-luminosity relation, were: $m-M=22.3$; distance= 930,000 light-years. The difference is due almost entirely to the revised form of the period-luminosity curve in the portion covering the Cepheids in the spiral.

zero-point of Shapley's period-luminosity curve.[1] It is hoped that the probable error arising from the system of magnitudes and the comparison with the Cepheids in the Magellanic Cloud is not greater than 0.2 mag. in the modulus, or 10 per cent in the distance.

Hitherto, wherever Cepheids have been found in isolated systems, the criterion has led to distances entirely consistent with other characteristics that could be observed. The latest application has been to N.G.C. 6822, a system very similar to the Magellanic Clouds,

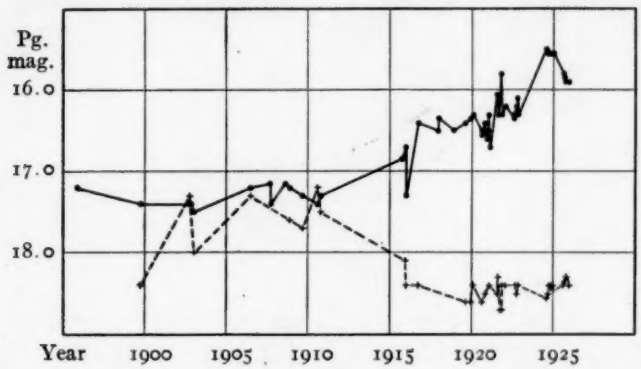

FIG. 4.—Two irregular variables in Messier 33; No. 1, crosses; No. 2, points

but at a distance of about 214,000 parsecs, comparable with that of Messier 33 itself.[2] Here, also, the period-luminosity relation functioned normally, and the order of the resulting distance was confirmed by several independent criteria. A presumption is thus raised in favor of the general validity of the Cepheid criterion which only strong and cumulative evidence to the contrary will destroy.

IRREGULAR VARIABLES

The four irregular variables are Nos. 1, 2, 6, and 13. A number of observations of Nos. 1 and 2 from early plates, not included in Table I, were published by Duncan[3] and by Wolf.[4] These, together with a

[1] Recent results on radial velocities and proper motions of galactic Cepheids indicate the possibility of a moderate correction to the zero-point, but until the new data have been discussed in a definitive manner, it seems preferable to use the current value, with full realization, however, of the possibility of a subsequent revision

[2] *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, **62**, 409, 1926.

[3] *Publications of the Astronomical Society of the Pacific*, **34**, 290, 1922.

[4] *Astronomische Nachrichten*, **217**, 476, 1923.

few additional data, are listed in Table III. The magnitudes have been reduced to the photographic system derived from comparisons with Selected Areas Nos. 21 and 45, and differ appreciably from those

TABLE III

ADDITIONAL MAGNITUDES FOR VARIABLES NO. 1 AND NO. 2

DATE	Obs.†	VARIABLE	
		No. 1	No. 2
1895 Nov. 14*	R	17.3‡	17.2
1899 Sept. 12*	K	18.4	17.4
1902 Sept. 5*	Ri	17.3	17.4
Sept. 26	W	.....	17.4
1903 Jan. 16*	R	18.0?	17.5
1906 *	L	17.3?	17.2
1907 Aug. 8	W	.....	17.15
Aug. 12	W	.....	17.4
1908 July 25	W	.....	17.15
Nov. 17	P	17.6	17.2
1910 Oct. 24	C	17.5	17.3
1915 Oct. 1*	M	.....	16.85
Oct. 30*	L	18.1	16.7
Nov. 5	C	18.4	17.3
1916 Sept. 28*	L	18.4	16.4
1917 Dec. 10	W	.....	16.5
Dec. 16	W	.....	16.35
1918 Nov. 11*	M	.....	16.5
1920 Aug. 15	W	.....	16.55
Oct. 4*	M	.....	16.4
1921 Aug. 30	Lu	18.3	16.3
Nov. 28	Lu	18.4	16.3
1922 Jan. 20	Lu	18.4	16.2
Aug. 14	W	.....	16.35
Aug. 25	W	.....	16.35
Aug. 25*	D	18.4	16.3
Oct. 14*	M	.....	16.1
Oct. 15*	M	.....	16.3

* Plates from which magnitudes have been independently determined. Otherwise the magnitudes are those published by Duncan and by Wolf, reduced to the system of the international photographic scale.

† The photographs were made by the observers indicated in the second column: R = Isaac Roberts, K = Keeler, Ri = Ritchey, W = Wolf, L = Lampland, P = Perrine, C = Curtis, M = van Maanen, Lu = Lundmark, D = Duncan.

‡ Limit of the plate. Variable No. 1 was not visible.

published by the above-mentioned investigators.[1] Independent estimates of the magnitudes have been made on such of the original plates as were available.

[1] Magnitudes of a sequence of comparison stars were derived from four short-exposure comparisons with Selected Area No. 45 in addition to those found from the available long exposures.

Light-curves of these two variables, based on Tables I and III, are shown in Figure 4. Number 2 appears to have remained quiescent at about 17.4 mag. from 1895 to somewhere between 1911 and 1915. By 1915, the variable had entered upon a period of activity which is still in progress. It brightened gradually, with minor fluctuations, until it reached a maximum of 15.55, which it maintained during the season 1924–1925. It was then the brightest star which could definitely be assigned to the nebula, the absolute photographic magnitude corresponding to the distance indicated by the Cepheids being −6.55. A faint slitless spectrogram was obtained during this period with two prisms and a 3-inch camera at the Newtonian focus of the 100-inch reflector. The continuous spectrum held up well into the violet, suggesting an early type, and a bright $H\beta$ could be seen. Several other bright lines, including $H\gamma$ and $H\delta$, were suspected. The spectrum can best be described as an early type with bright lines, and this is consistent with the photovisual plates which indicate a very small color-index. During the summer and autumn of 1925, the star faded about 0.5 mag. Variation of this type is not sufficiently known among galactic stars for analogies to be traced.

Variable No. 1 is similar in many respects to No. 2. Its normal magnitude appears to be near 18.4, but about 1900 it entered upon a seventeen-year period of activity, during which it attained a maximum magnitude of at least 17.2. By 1917 it was back to normal, and since then has shown only slight evidence of unrest. The mean magnitude during 1924–1925, however, was 18.3 as against 18.5 for the period 1920–1922. Photovisual observations indicate a moderate positive color-index of the order of 0.5 mag.

Variable No. 6 has remained steady at about 18.8 since 1921. Plates for 1910, 1911, and 1919 indicate, however, a luminosity some 0.3 or 0.4 mag. brighter. Variable No. 13 is fainter on the early plates than on the later. In 1910–1911 it was about 18.4; in 1919–1921, about 18.0; in 1924–1925, about 17.7. The color-index during the last epoch was of the order of +0.8 mag.

OTHER VARIABLES

Variable No. 8 appears to be an eclipsing star. On three plates it was 19.1 or fainter; otherwise it has remained steady at about 18.6. The minima were

	Photographic Magnitude
J.D. 2424056.49	19.15
4377.59	19.1
4387.60	19.25

The images on these plates have been critically examined, and there seems to be no reason for supposing them defective.

No periods have been found for variables Nos. 14, 15, 21, 32, and 45. The small amplitudes and recurring maxima suggest, however, the Cepheid type of variation, and it is probable that more and better observations will establish it for several of the stars.

NOVAE

Two novae have been observed in the spiral, the first about 1′.8, and the second about 10′.8, from the nucleus, both in the north following quadrant. Nova No. 1 was found on four plates obtained by Pease in the winter of 1919–1920. The position, indicated on Plate XV, was about 1″.5 south of a star of 18.15 mag., and the photographic images of the two overlapped. This detracted from the reliability of the estimates of magnitude. Moreover, an interval of one hundred and ten days separated the plate on which the nova was first seen from the next preceding plate, and thus the date and the magnitude of maximum are unknown.

Nova No. 2, whose position is marked on Plate XIV, appeared on the first photograph obtained in the season 1925–1926. It was obviously on the declining branch of its light-curve, although it was followed for some two months longer. The data of observation for both novae are given in Table IV, in which are included absolute magnitudes computed from the distance indicated by the Cepheids, and also the dates of the photographs next preceding and following those on which the novae were visible.

The observed portions of the two light-curves are shown in Figure 5. The data are insufficient to serve as a reliable criterion of

distance, and would be so even were the frequency distribution of absolute magnitudes attained by galactic novae definitely known. Among those in Messier 31, however, of which some sixty have been observed,[1] there is a sharply defined maximum frequency about

TABLE IV

APPARENT AND ABSOLUTE PHOTOGRAPHIC MAGNITUDES OF NOVAE IN MESSIER 33

PLATE	DATE	m	M†
Nova No. 1			
D 34*	1919 Aug. 26	.....	.....
P 133	Dec. 14	17.25	−4.85
134	Dec. 18	17.25	4.85
138	Dec. 21	17.8	4.3
139	1920 Jan. 23	18.0	−4.1
D 48*	Aug. 13	.....	.....
Nova No. 2			
H 517*	1925 Jan. 17	.....	.....
577	July 17	17.9	−4.2
D 253	July 20	18.0	4.1
H 584	July 26	18.4	3.7
588	Aug. 15	19.2	2.9
600	Aug. 17	19.25	2.85
D 277	Aug. 21	19.4	2.7
288	Aug. 24	19.5	2.6
H 606	Sept. 12	19.5	2.6
611	Sept. 21	19.7	−2.4
S 507*	Oct. 18	.....	.....

* The photographs next preceding that of the first appearance and next following the last appearance are also listed.

† Absolute magnitudes are derived from the distance indicated by the Cepheids in Messier 33.

apparent magnitude 17.0. The Cepheids in the two spirals have the same apparent magnitudes for corresponding periods, and it is to be expected that the novae, also, will show a similar behavior. The fragmentary observations of the two novae in Messier 33 are consistent with this supposition. They indicate, at least, that the novae are brighter than the Cepheids, a relation that holds in both Messier 31 and in the galactic system. The lack of data on the magnitudes at maxima is a serious handicap, but on the very reasonable

[1] This number includes those discovered up to December, 1925. Nos. 23–60 have been found by the writer; details will be published later.

assumption that the apparent magnitudes attained in Messier 33 are similar to those in Messier 31, the mean absolute magnitude of the novae, at the distance indicated by the Cepheids, would be -5.1, which is of the same order as that for novae in the galactic system.[1]

DIFFUSE NEBULAE IN MESSIER 33

Several patches of diffuse nebulosity with stars involved are found in the outer arms of Messier 33. The brightest of these, cata-

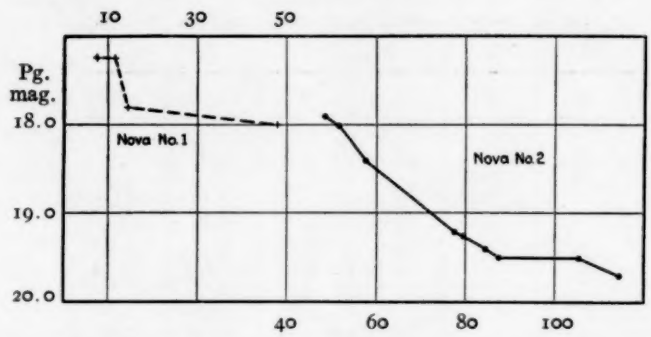

FIG. 5.—Light-curves of two novae in Messier 33. The Julian dates may be found by adding to the abscissae, 2422300 for Nova No. 1, and 2424300 for Nova No. 2.

logued separately as N.G.C. 588, 595, and 604, show emission spectra. Slit spectrograms are available for N.G.C. 604, and show that $H\beta$ is brighter than N_2, a relation characteristic of diffuse emission nebulosity in the galactic system. Photovisual observations indicate that the brighter stars involved are blue.[2] Moreover, as will be shown later, the relation between the apparent magnitudes of the involved stars and the angular extent of the nebulosities is the same as that among diffuse galactic nebulae.[3]

These various features offer strong cumulative evidence that the patches in Messier 33 are indeed diffuse emission nebulae similar in

[1] Lundmark (*Monthly Notices*, **85**, 885, 1925) has made use of the reverse procedure. Combining a mean apparent magnitude of 16.34 for twenty-two novae in Messier 31 with a mean absolute magnitude of -6.9 for galactic novae, he derived a distance of 1.4×10^6 light-years for the spiral.

[2] The photovisual images are as small as, or smaller than, those of any other stars in the nebula of the same order of photographic magnitude.

[3] *Mt. Wilson Contr.*, No. 250; *Astrophysical Journal*, **56**, 400, 1922.

every respect to those in the galactic and other systems. On this assumption, the brighter stars involved are blue giants[1] of spectral types Bo or Oe5, and can be used to test the order of the distance derived from the Cepheids. The photographic magnitudes of the brighter stars in the three groups[2] are given in Table V along with the absolute magnitudes computed from the modulus $m-M=22.1$.

TABLE V*

MAGNITUDES OF STARS INVOLVED IN EMISSION NEBULOSITY IN MESSIER 33

N.G.C. 588		N.G.C. 595		N.G.C. 604	
m	*M*	*m*	*M*	*m*	*M*
16.55	−5.55	16.6	−5.5	15.75	−6.35
16.85	5.25	16.7	5.4	15.8	6.3
17.5	4.6	16.8	5.3	15.9	6.2
17.7	−4.4	16.9	5.2	15.95	6.15
		17.0	5.1	16.0	6.1
		17.2	4.9	16.15	5.95
		17.5	4.6	16.15	5.95
		17.6	4.5	16.3	5.8
		17.6	4.5	16.45	5.65
		17.7	−4.4	16.55	5.55
				16.55	5.55
				16.75	5.35
				16.8	5.3
				16.85	−5.25

* The table contains photographic magnitudes, both apparent and absolute, of the brighter stars involved in three patches of diffuse nebulosity having emission spectra, in the arms of Messier 33. The lists are complete to the faintest star measured in each group.

These absolute magnitudes must be compared with those of the brightest blue giants known in other systems. The most recent estimates of mean absolute photographic magnitude for the blue giants in the galactic system are

Bo (sharp lines) $=-3.5$ (Adams)[3]
O (absorption lines) $=-4.4$ (Plaskett)[4].

[1] *Mt. Wilson Contr.*, No. 241; *Astrophysical Journal*, **56**, 162, 1922.

[2] N.G.C. 592, the only other knot in the arms of Messier 33 which is catalogued separately, is an open group of stars with no trace of diffuse nebulosity. One star, about 15.5 and thus considerably brighter than the others, is red and may be a field star. The three next brightest, ranging from about 16.5 to 16.8, are of early types, blue or white.

[3] *Mt. Wilson Contr.*, No. 262; *Astrophysical Journal*, **57**, 294, 1923.

[4] *Publications of the Dominion Astrophysical Observatory*, **2**, No. 16, 1924.

The original values, expressed in visual magnitudes, have been corrected for a mean color-index of −0.4 mag. in each case. Plaskett suggests a dispersion of the order of 3 mag. for the O stars, which would make the upper limit about −5.9, less than a half-magnitude fainter than the brighʿest stər in Table V. When we consider the influence of selection on the stars in the spiral, the probability that some of the brightest images are clusters rather than single stars,[1] and the uncertainty in the dispersion among the galactic stars, the discrepancy becomes unimportant, so that the two sets of absolute magnitudes may be regarded as comparable. The blue giants in Messier 33 therefore confirm the order of the distance derived from the Cepheids.

Dimensions of the nebulosities themselves, measured on long exposures with the 100-inch reflector, together with the linear dimensions in parsecs are as follows:

	Seconds of Arc	Parsecs
N.G.C. 588	30×25	40×30
595	55×32	70×40
604	65×55	80×70

The brighter portion of N.G.C. 604, in which the stars are closely crowded, is about 50″, or 65 parsecs, in diameter. These dimensions are comparable with those of the largest nebulae in other systems. N.G.C. 2070 in the large Magellanic Cloud is about 80 parsecs;[2] nebula No. III in N.G.C. 6822 is about 55 parsecs;[3] and in the galac-

[1] Groups of stars in Messier 33 one second (1.27 parsecs) or less in diameter would certainly appear as ordinary images of single stars on all but limiting exposures under excellent observing conditions, and hence would be counted as single stars. Groups of such dimensions are fairly numerous in the galactic system, especially among the blue giants involved in nebulosity. Within this limit, for instance, are included such aggregations as five of the six brightest stars in the Pleiades, all the brighter stars in the Orion nebula, the major portions of the clusters in Messier 8 and N.G.C. 2237, and such multiple stars as those in Messier 20 and N.G.C. 281. Phenomena of a similar nature would readily account for several of the excessively bright objects listed as single stars in Messier 33, especially among those in the crowded cluster involved in N.G.C. 604. The excess of very bright stars, all apparently blue, over those to be expected from an analogy with Kapteyn's luminosity function also can be explained, in part at least, along these lines.

[2] Shapley, *Harvard College Observatory Bulletin*, No. 816.

[3] *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, **62**, 409, 1926.

tic system the great exterior loop in the constellation of Orion is about 40 parsecs in diameter.[1]

Among the diffuse nebulae in the galactic system, the following relation has been established for measures on Seed 30 plates exposed sixty minutes with a reflector of focal ratio 1:5,

$$m+5 \log a = 11.1$$
$$\text{Mean deviation} = 0.8,$$

where m is the apparent photographic magnitude of the associated star or group of stars and a is the angular distance, in minutes of arc, to which the luminous nebulosity can be traced.[2] For the nebulae in Messier 33, using integrated magnitudes from Table V and measuring the extent of the nebulosity from the center of gravity of the stars, we find

	m	a	$m+5 \log a$
N.G.C. 588	15.5	0'.15	11.4
595	14.6	0.33	12.2
604	13.2	0.5	11.7
Mean	.....	.....	11.8

These results are sufficiently consistent with the foregoing relation to support the assumption that the stars and nebulae in the spiral are similar to those in the galactic system.

LUMINOSITY FUNCTION

In order to determine the upper limit of luminosity for the stars in Messier 33, counts were made over three areas, each about six minutes of arc square, centered on the nucleus and on points six minutes north and eight minutes south, respectively. The sum of the areas is 107 square minutes of arc, which was estimated to contain about one-fifth of the stars in the nebula. The same method

[1] Measured on long exposures with small cameras. The original plates are in the Mount Wilson collection.

[2] *Mt. Wilson Contr.*, No. 250; *Astrophysical Journal*, 46, 400, 1922.

was employed as in making counts[1] in N.G.C. 6822, the final results being expressed as numbers of stars in successive intervals of apparent photographic magnitude. Field stars were eliminated by means

TABLE VI

LUMINOSITY FUNCTION FOR MESSIER 33

Pg. Mag.	Stars Counted	Normal Number Gal. Lat. 30°.5	$\phi(m)$* Messier 33	$\phi(M)$† Galaxy	M
19.2–19.0	990	16	974	972	−3.0
18.9–18.7	540	14	526	580	3.3
18.6–18.4	340	12	328	341	3.6
18.3–18.1	265	10	255	197	3.9
18.0–17.8	233	9	224	114	4.2
17.7–17.5	110	8	102	64	4.5
17.4–17.2	45	7	38	35	4.8
17.1–16.9	29	6	23	19	5.1
16.8–16.6	25	5	20	10.4	5.4
16.5–16.3	17	4.3	12.7	5.5	5.7
16.2–16.0	10	3.5	6.5	2.9	6.0
15.9–15.7	4	2.9	1.1	1.5	6.3
15.6–15.4	6	2.4	3.6	0.8	6.6
15.3–15.1	2	2.0	0	0.4	6.9
15.0–14.8	3	1.7	1.3	0.2	7.2
14.7–14.5	1	1.4	−0.4	0.1	−7.5
14.4–14.2	1	1.1	− .1	.....	.....
14.1–13.9	1	.9	0.1	.....	.....
13.8–13.6	2	.7	1.3	.....	.....
13.5–13.3	2	.5	1.5	.....	.....
13.2–13.0	2	.4	1.6	.....	.....
12.9–12.7	0	.3	−0.3	.....	.....
12.6–12.4	0	.2	− .2	.....	.....
12.3–12.1	0	.2	−0.2	.....	.....

* $\phi(m)$ represents the numbers of stars in successive intervals of apparent photographic magnitude in a portion of Messier 33. The intervals are 0.3 mag., and the area over which the counts were made is estimated to include about one-fifth of the stars in the nebula.

† $\phi(M)$ gives the numbers of stars in successive intervals of absolute visual magnitude in a volume of space 3.6×10^8 cubic parsecs in which stars are distributed according to Kapteyn's luminosity function for the vicinity of the sun. The intervals are the same as for $\phi(m)$, namely, 0.3 mag. The median absolute magnitudes of the intervals in both functions are given in the last column. These are visual for Kapteyn's function and photographic for that of the nebula. The latter absolute magnitudes are based on the distance indicated by the Cepheids.

of the stellar distribution tables recently published by Seares and van Rhijn.[2]

The data are collected in Table VI and are represented diagrammatically in Figure 6. The results for the brighter stars have little weight because of the small numbers in so restricted an area,

[1] *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, **62**, 409, 1926.

[2] *Proceedings of the National Academy of Sciences*, **11**, 358, 1925.

but it is evident[1] that the stars in the nebula begin about 15.5 mag. For the distance derived from the Cepheids, this corresponds to an absolute magnitude of −6.6.

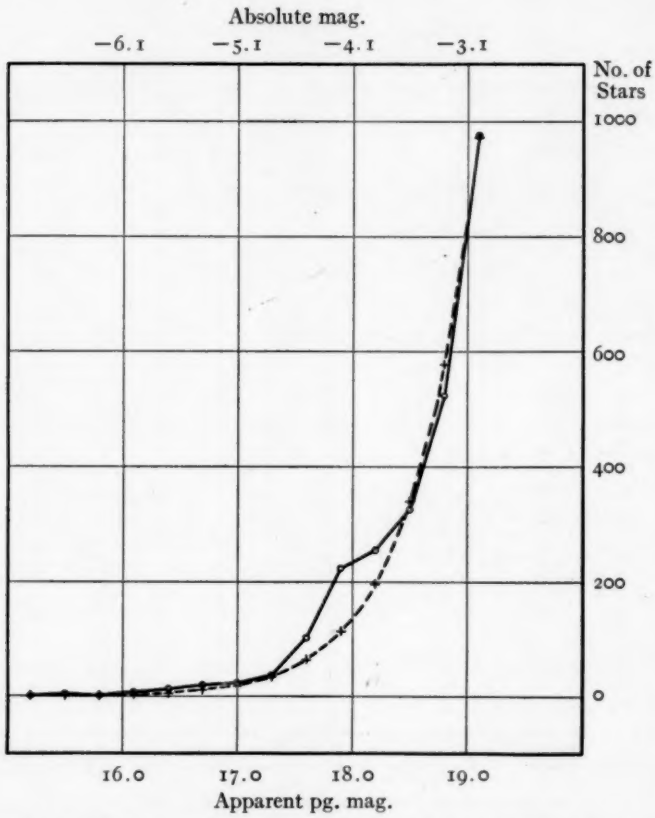

FIG. 6.—Luminosity function for the brighter stars in Messier 33, compared with Kapteyn's luminosity function.

Such a value for the upper limit of stellar luminosity is consistent with the conception of Messier 33 as a great isolated system, and is

[1] There are about six stars between 13.0 and 15.5 in excess of the number to be expected from the distribution tables. These are readily explained as accidental deviations incident upon the restricted areas counted, or resulting from the counting of close groups as single stars; but it is possible that they may indicate the presence of a few extremely luminous stars in the nebula. More extensive counts will be required in order to settle the question.

comparable with the corresponding limits for other systems.[1] The brightest Cepheid, at maximum, is about 2.5 mag. fainter than this limit and, indeed, has been outshone by several of the irregular variables. One of the latter, No. 2, practically reached the limit when, during the recent maximum, it attained an absolute magnitude of −6.55. Among the stars of constant luminosity, the five brightest of the blue giants in N.G.C. 604 are all within a half-magnitude of the limit.

The nucleus, if it were truly stellar, would be an exception. The apparent magnitude is about 14.1, or −8.0 absolute. Since the spectral type is G5, the absolute visual magnitude, −9.0, would rival that of S Doradus in the large Magellanic Cloud. The image of the nucleus as shown on short exposures is not sharp, however, and suggests a small, highly concentrated globular nebula rather than a star. This is consistent with the behavior of nuclei in other spirals, among which the only ones that appear to be truly stellar have planetary spectra.[2]

The plot of the luminosity function, shown in Figure 6, exhibits a striking similarity to the corresponding section of Kapteyn's luminosity curve for stars in the vicinity of the sun, even though the latter is expressed in visual magnitudes, while the former is photographic. Kapteyn's function[3] is drawn as a dotted curve. The minor discrepancies between this and the full broken line, representing the data for Messier 33, can be explained partly as due to clusters in the nebula having been counted as single stars of higher luminosity and partly on the assumption, for which observational evidence will be adduced in the following section, that the brightest stars in the nebula are blue, and that the mean color-index increases as the magnitudes grow fainter.

[1] Upper limits of luminosity for other systems, in photographic magnitudes, are as follows: galactic system, −5.5 (brightest reliable measure, β Orionis); large Magellanic Cloud, −8.0 (*Harvard Circular*, No. 271); small Magellanic Cloud, −4 to −7 (*ibid.*, No. 260); N.G.C. 6822, −5.9 (*Mt. Wilson Contr.*, No. 304); and globular clusters, −4.0 (*Proceedings of the National Academy of Sciences*, **6**, 293, 1920).

[2] N.G.C. 4051 and 4151 are examples. The emission spectra are confined to the nuclei and do not represent the light from the body of the nebulae.

[3] The data are found in *Mt. Wilson Contr.*, No. 229; *Astrophysical Journal*, **55**, 242, 1922.

Since the star counts are estimated to include about one-fifth of all the stars in the nebula down to the limit of the counts, the volume coefficient applied to Kapteyn's function as shown in Figure 6 must be increased fivefold in order to include all the stars in the nebula in the successive intervals of absolute magnitude. The resulting value is 1.8×10^9 cubic parsecs.

The integration of Kapteyn's function over the entire range of magnitudes gives a space density of 8.16 absolute visual magnitudes per cubic parsec.[1] For the volume coefficient corresponding to the counts in Messier 33, the total magnitude would be

$$8.16 - 2.5 \log (1.8 \times 10^9) = -15.0 \text{ (absolute)}$$
$$22.1 - 15.0 = \quad 7.1 \text{ (apparent)} .$$

The apparent visual magnitude of the spiral has been measured by Holetschek[2] and by Kritzinger,[3] and both find the same value, 7.0. This excellent agreement immediately suggests that the similarity between Kapteyn's luminosity function and that for the stars in Messier 33, although actually observed for only the three brightest magnitudes, extends throughout the range of magnitudes significant in determining integrated luminosities. The lower limit of this range is about the fourth absolute magnitude.[4] Fainter stars contribute less than 0.2 mag. to the total luminosity.

[1] Seares, *Mt. Wilson Contr.*, No. 191; *Astrophysical Journal*, **52**, 162, 1920.

[2] *Annalen der Wiener Sternwarte*, **20**, 1907.

[3] *Sirius*, **48**, 111, 1915.

[4] In a system of stars whose luminosities are distributed according to Kapteyn's function, the luminosity contributed by stars fainter than magnitude M, expressed as a correction which must be applied to the result of integration over the range $-\infty$ to M in order to obtain the total magnitude of the entire system, is as follows:

M	Correction	M	Correction
−3	3.52 mag.	+3	0.32 mag.
2	2.65	4	.17
−1	1.93	5	.08
0	1.35	6	.036
+1	0.90	7	.013
+2	0.56	+8	0.004

For stars fainter than about $M = 8$, Seares (*Mt. Wilson Contr.*, No. 273; *Astrophysical Journal*, **59**, 310, 1924) has shown that Kapteyn's function does not represent the observed distribution in the vicinity of the sun. Probable differences for this part of

The total luminosity of the spiral is thus fully accounted for on the assumption that it is a system of stars, the ten brightest magnitudes of which are distributed in a manner similar to that observed in the neighborhood of the sun. This assumption is supported by the directly observed similarity in the distribution of the three brightest magnitudes. It is tempting to extrapolate and argue that, since the ten brightest magnitudes appear to be similarly distributed in the two systems, the fainter magnitudes also should conform. There are, however, no observational controls for such a venture. Even the first assumption is by no means established. For instance, it might well be suggested that stars beyond the limits of observation are entirely lacking and that the luminosity which would otherwise be derived from them is supplied by true nebulosity which would ultimately produce the stars. This, of course, would be a forced interpretation, but would conform, in a measure, to current speculations on the evolution of spirals.

The volume of space filled by Messier 33 can be approximately derived by considering it as an ellipsoid of revolution with the minor axis one-tenth the major axis. The true form is probably a lens shape, but the order of the volume can be obtained on the simpler assumption. Since the maximum diameter is about sixty minutes of arc and the linear scale, at the distance indicated by the Cepheids, is $1'' = 1.27$ parsecs, the volume will be of the order of

$$\tfrac{4}{3}\pi(2.3)^3 \times 10^8 = 5 \times 10^9 \text{ cubic parsecs.}$$

This is about three times the volume coefficient derived by adjusting Kapteyn's luminosity function to fit the star counts in the nebula. It indicates that the average space density for the nebula as a whole is about one-third that in the vicinity of the sun, or about 9.26 absolute visual magnitudes per cubic parsec. The space density in the nuclear region and in the arms would, of course, be much greater.

An approximation for the mass of the spiral can be obtained from the difference in the radial velocities of the nucleus and the patch

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the curve have, however, no appreciable effect on the integrated luminosity. When stars no fainter than $M = 15$ are considered, the radically different form of the function which Seares has suggested for the faint stars corresponds to a difference in the integrated luminosities of less than 0.002 mag.

of emission nebulosity, N.G.C. 604, some 700″ from the nucleus in the general direction of the major axis. The measures are

	Km/sec.
Nucleus[1]	− 70
N.G.C. 604[2]	−270
Difference	200

The ratio of minor to major axis appears to be about 2:3; the tilt is therefore about 42°, and the differential velocity of 200 km/sec. can be interpreted as the radial component of a linear velocity of rotation about the nucleus of

$$200 \sec 42^\circ = 270 \text{ km/sec.}$$

This corresponds to a period of rotation of the order of twenty million years at a distance of 1.83×10^8 astronomical units from the nucleus. The mass within this orbit, in terms of the sun's mass,[3] will be

$$M = \frac{a^3}{p^2} = 1.5 \times 10^{10} .$$

The additional mass for the outer regions is probably less than the errors involved in the determination.

The mean density on these assumptions is approximately $3\odot$ per cubic parsec, or about twenty times that in the vicinity of the sun. This suggests the possibility of large quantities of non-luminous material in the central region of the nebula, but until more accurate data are available no emphasis can be placed on the numerical values. The method employed in calculating the mass is very sensitive to errors in the radial velocities—the mass varying directly with the square of the linear velocity of rotation. The radial velocity

[1] Pease, *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

[2] Mean of two determinations, by Pease and by Slipher. One, −278 km/sec., is given in the paper just cited; the other, −260 km/sec., is quoted by Strömberg, *Mt. Wilson Contr.*, No. 292; *Astrophysical Journal*, **61**, 353, 1925.

[3] When the distance from the nucleus, a, is expressed in parsecs, and the linear velocity of rotation, v, in km/sec., then $M\odot = 235\, av^2$. The mass varies directly as the distance from the observer and as the square of the linear velocity. Due to the uncertainty in the latter quantity and the possibility that something more is observed than simple rotation around the nucleus, the computed mass may be in error by as much as a factor of 10.

of N.G.C. 604 was derived from narrow emission lines and should be fairly reliable. That for the nucleus,[1] however, depends upon three absorption lines measured on a single plate of very small dispersion, −0.9 mm from $H\gamma$ to K. The result represents only the order of the velocity, and the probable error is a large fraction of the difference between the velocities of the nucleus and N.G.C. 604. Moreover, it is by no means certain that even an accurate differential velocity would represent merely the line-of-sight component of rotation about the nucleus.

COLORS OF STARS IN MESSIER 33

Some information concerning the colors of the brighter stars in Messier 33 has been derived from isochromatic plates exposed through a visual color-filter. One of these, an exposure of three hours with the 100-inch reflector, shows stars which are estimated to be fainter than the eighteenth photovisual magnitude, including many of the Cepheids. Comparisons on the blink-comparator with appropriate direct exposures suggest that most of the brighter stars in the nebula are white or blue. The conclusion is based on the differences in the photovisual images of stars of approximately equal photographic magnitudes. For example, in the central region of the nebula, twenty-eight stars have photographic magnitudes between 16.5 and 17.5. On the plates examined, twenty-five of these show photovisual images that are comparable with the photographic images, while the remaining three stars have much larger photovisual images. From the well-determined photographic scale, which in this region appears to be not very different from the photovisual, the color-indices of the three outstanding stars were estimated to be about a magnitude greater than those of the other twenty-five. Moreover, two faint stars in the immediate neighborhood, having conspicuous proper motions,[1] are obviously much redder than the

[1] Six stars within 11′ of the nucleus show conspicuous proper motions on the blink-comparator on plates of fifteen years' interval made with the 60-inch reflector. The photographic magnitudes of two have been measured as 17.3 and 18.25, the proper motions being of the order of 13″ and 10″ per century, both toward the south-following quadrant. The brighter of these is No. 367 in van Maanen's measures of Messier 33 for rotation (*Mt. Wilson Contr.*, No. 260; *Astrophysical Journal*, **57**, 264, 1923). The magnitudes of the other four are estimated to be 14.2, 15.8, 16.4, and 17.8, with proper motions between 5″ and 10″ per century.

three just mentioned. The most reasonable interpretation of these observations is that the proper-motion stars are red dwarfs, that the three exceptional stars are yellow, and that the twenty-five other stars are white or blue. Moreover, since faint stars of early spectral types are extremely rare in this galactic latitude, it is probable that such stars belong to the nebula.

Although it has not yet been possible to calibrate the photovisual plates by direct comparisons with the polar sequence, an approximate reduction curve, connecting photovisual magnitudes with scale measures, has been constructed by correcting the photographic magnitudes for color-indices corresponding to known or probable spectral types. For the bright end of the curve, the nucleus itself and a star 1'.8 north following were used. Photovisual magnitudes of 13.1 and 12.3, respectively, were derived from the observed spectral types G5 and F8. Seven Cepheids gave a point for the faint end at about 18.3. A mean spectral type of Go was assumed, in accordance with data from spectrograms of sixteen typical galactic Cepheids having periods ranging from ten to twenty-seven days with a mean of about seventeen days. The mean period of the seven Cepheids in Messier 33 is twenty-three days, but this larger value is balanced, to a certain extent, by the fact that most of the Cepheids were observed near maximum. The midpoint of the curve, the most important for the present purpose, depends upon a single star. This has a proper motion of the order of 10″ per century and a photographic magnitude of 18.25. It is obviously very red, and has been assumed to be a dwarf M with a color-index of +1.8. It is improbable that the color-index is greater than this amount, and hence the photovisual luminosities derived from this region of the reduction curve, together with the resulting color-indices, can be considered as upper limits.

By means of the reduction curve, color-indices have been determined for one hundred and ten stars, all but one[1] of which are within 2′ of the nucleus. The results are exhibited in Figure 7, where color-indices are plotted against photographic magnitudes. The data are practically complete to the eighteenth magnitude. The dotted

[1] The exception is the faint proper motion star used in determining the reduction curve. It is about 2'.2 south preceding the nucleus, and is identified on Plate XV.

curve represents the minimum values of color-indices for stars of corresponding magnitudes in the polar sequence.[1] Since the galactic latitude of the pole is only 2° less than that of the spiral, it is reasonable to assume that field stars will be confined to the region above the dotted line in Figure 7. This region includes nine stars brighter than the eighteenth magnitude, one of which, the irregular variable No. 13, 17.85 mag., +0.8 color-index, certainly belongs to the nebula. The number of field stars to be expected on the basis of the mean distribution tables of Seares and van Rhijn[2] is seven.

The restricted area and the small number of stars do not permit definite conclusions to be drawn from this close agreement. The in-

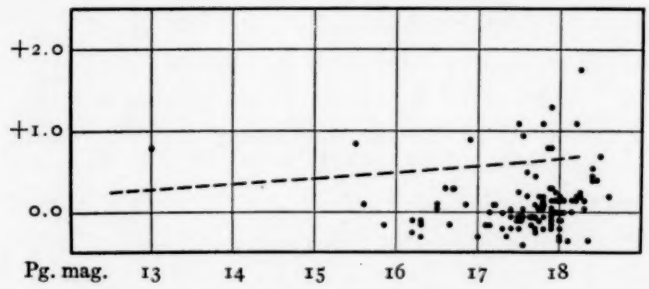

FIG. 7.—Color-indices (ordinates) of stars in the central region of Messier 33. The dotted line indicates the probable minimum color-index of stars outside the nebula.

dications are, however, that all the stars brighter than the eighteenth magnitude, having color-indices less than, say 0.5 mag., belong to the nebula. Moreover, no star, excepting the variable, brighter than this limit, with a color-index greater than 0.5 mag., can with certainty be assigned to the central region of the nebula. The mean color-index of the stars assigned to the nebula is slightly negative, but since this probably represents an upper limit, the stars may actually be blue giants. It is possible that the brightest red stars in the spiral are all variables.

These results, although approximate and possibly subject to serious errors and limitations, are sufficient to differentiate the brighter stars in the central region of Messier 33 from those in the globular

[1] Seares, *Mt. Wilson Contr.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

[2] *Mt. Wilson Contr.*, No. 301; *Astrophysical Journal*, **62**, 320, 1925.

clusters and in the irregular nebulae such as N.G.C. 6822, and probably from those in the Magellanic Clouds, all of which appear to be red. This contrast, which may be important, invites speculation on the larger problems of cosmic evolution.

In this connection, attention may be called to Seares's results[1] on the distribution of colors in Messier 51, 94, and 99. These are all well-developed spirals in which the arms are partially resolved into stars. The central regions of unresolved nebulosity (and also the amorphous mass at the end of an arm of Messier 51) were found to be much redder than the outer arms. Messier 33 exhibits the same phenomenon. The unresolved nebulosity around the nucleus is much stronger on photovisual than on unfiltered plates, as compared with star images both in the central region and in the outer arms. This agrees with the fact that the nucleus and the surrounding remnant of apparently unresolved nebulosity give a spectrum of the solar type, while the brighter stars have no appreciable color-index. The contrast in color between stars and nebulosity is in harmony with Seares's results for the other spirals, which in turn suggest that there is no conspicuous difference in color between the brighter stars in the central region and those in the outer arms of spirals. An inspection of the images in the arms of Messier 33 appears to support this latter suggestion, but, since the photovisual exposures were centered on the nucleus, the aberrations of the images in the outer regions prevented any reliable quantitative conclusions.

The observational data are insufficient to determine whether the unresolved central nebulosity with its solar type spectrum and considerable color-index is actually amorphous nebulosity or a cluster of faint stars. If the latter is the case, the color-indices must be assumed to increase as the luminosities of the stars decrease, after the manner of stars in the galactic system. It is well known, however, that spirals can be arranged in a series in which the arms build up at the expense of the unresolved nuclear region, unwinding and breaking up into condensations as they grow. The nuclear region of such late-type spirals as Messier 33 can be thus considered as the remnant of much larger central masses, such, for instance, as seen in Messier 31 or N.G.C. 4594. The nuclear regions of these in turn are

[1] *Proceedings of the National Academy of Sciences*, **2**, 553, 1916.

almost indistinguishable from the elliptical nebulae (possibly pre-spiral forms) such as N.G.C. 3115 and Messier 32. In all these cases, the apparently unresolved nebulosity is associated with approximately solar-type spectra and appropriate color-indices. Even in Messier 31, however, which is highly resolved in the outer arms, the nuclear region shows not the slightest trace of resolution with the 100-inch reflector working under exceptionally fine observing conditions.

CONCLUSION

The present investigations indicate that Messier 33 is an extremely distant system of stars and nebulae, similar in many respects to the Magellanic Clouds. It differs, however, in two important characteristics: The great majority at least of the brightest stars are of early spectral types, and there is a symmetrical arrangement of material with respect to a dominating nucleus. Detailed study of the stars within the limits of observation—the three or four brightest magnitudes—indicates that they are normal giants such as are known in other systems. The application of all available criteria leads to this one interpretation of the data.

The distance, derived from the period-luminosity relation applied to thirty-five apparently normal Cepheids, is about 8.1 times that of the small Magellanic Cloud, or, using Shapley's distance for the latter, about 263,000 parsecs. The order of this distance is confirmed by evidence from independent sources—novae, emission nebulosity with blue stars involved, and the luminosity function. The distance being known, the apparent dimensions of the spiral can be converted into absolute dimensions, assuming the form, from analogy with other spirals, to be lenticular, with a minor axis about one-tenth of the major axis. These dimensions are listed in Table VII with those of the two Magellanic Clouds for comparison.[1]

It is evident that Messier 33 must be considered as comparable in size with the Clouds rather than with the galactic system. It is not, however, of an entirely different order from the latter, and, in this connection, it may be mentioned that in size the other great spiral, Messier 31, occupies an intermediate position. Including the Magel-

[1] *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, **62**, 409, 1926.

lanic Clouds, which are most readily interpreted as irregular non-galactic nebulae, five systems are now known which are believed to be distinct from the galactic system. The diameters range from

TABLE VII

ABSOLUTE DIMENSIONS OF MESSIER 33 AND THE MAGELLANIC CLOUDS

	MESSIER 33	MAGELLANIC CLOUDS	
		Large	Small
Distance*	263,000	34,500	31,600
Total magnitude	−15.1 pv.†	−16.5 pg.	−15.5 pg.
Diameter	4600	4300	2000
Volume	5×10^{9}	4.2×10^{10}	4.2×10^{9}
Density	9.3	10.0	8.5

* Distances and diameters are expressed in parsecs, volumes in cubic parsecs, and densities in absolute magnitudes per cubic parsec.

† The total magnitude of Messier 33 is expressed in visual absolute magnitudes, but, since the brighter stars in the spiral are white or blue, it is thought that the total photographic magnitude will not be very different.

1250 parsecs for N.G.C. 6822 to about 14,000 parsecs for Messier 31. The ratio is about 1 to 11. The multiplication of the dimensions of Messier 31 in the same ratio would include the galactic system, even as outlined by the globular clusters.

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REVIEWS

Foundations of the Universe. By M. LUCKIESH. New York: D. Van Nostrand Company, 1925. Pp. vii+245. Frontispiece, 17 Figures. $3.00.

The book, *Foundations of the Universe*, has a title that for obvious reasons commands attention. Whether such things are logically possible and can be adequately described are questions that Mr. Luckiesh has disposed of in a very satisfactory way. The tentative nature of the foundations is indicated in the Preface, but their soundness, breadth, and general character are made strikingly clear in the pages that follow.

The book presents in a condensed but lucid way the development of modern scientific theory concerning the structure and operation of the universe of matter. It is designed for the general reader and contains a minimum of technical language. The introductory chapter explains the slow progress of science before the time of Newton as the result of speculation insufficiently supported by experimentation and of an egoism that prevented open-minded investigation. Succeeding chapters review briefly the contributions of Copernicus, Kepler, Galileo, and Newton to astronomy and mechanics, of Rumford, Boyle, Brown, Maxwell, and Planck to the theory of molecular motion. The chapters dealing with the subject of light introduce the question of the aether, Maxwell's electro-magnetic theory, the fundamental character of the velocity of light, and the Einstein epoch.

The chemical phase of the subject receives consideration under the headings of atomic theory, isotopes, periodic table of elements, etc.

With chapter ix begins a discussion of the electron theory, inorganic evolution, quantum theory, properties of radio-activity. Chapter xiii is devoted to atomic structure and models by Thomson and by Bohr, and Somerfeld's modification in the application of the relativity theory. Balmer series and other matters of spectroscopic interest receive attention here.

The conclusion follows that scientific theory is now sufficiently sound and powerful to give a plausible explanation of the three baffling problems, transmutation of elements, sub-atomic energy, and stellar radiation, in fact as phases of one and the same process.

To the non-scientific reader, *Foundations of the Universe* presents in a very compact and effective way essential principles and their development. To a reader interested in science, but not working in this particular field, the book sums up in a systematic and clarifying way ideas that may have been previously obtained in a rather haphazard manner. To a student of physics, there is least of direct value on account of the necessary brevity of treatment, but even for this type of reader the book is valuable as an example of how involved technical theory can be summarized and made intelligible to the general reader.

It would be remarkable if such a large field could be covered without errors or inaccuracies. One such inaccuracy occurs in the statement of Newton's second law of motion on page 23, which should read, "rate of change of motion is proportional, etc.," instead of "change of motion." On pages 189–90 there seems to be some confusion in the use of letters *a* and *b* in the discussion of the Rydberg series. One wonders also why the determination of the charge of an electron was not mentioned in connection with the name of R. A. Millikan on page 235.

The illustrations and examples used are drawn from familiar material and are illuminating. The explanations are generally very clear and the statements of ideas show broad views effectively expressed. The statement concerning Newton and Einstein on page 27 will help to reduce the bewilderment of mind and lack of confidence that many readers have experienced in the apparently revolutionary ideas of recent years.

It is a fortunate circumstance that a book so entirely devoted to the theoretical should be written by one whose approach to the subject would be expected to be from the industrial side. It emphasizes the community of interest and the advantages of co-operation, and at the same time will extend the influence of the book among general readers.

In reading *Foundations of the Universe* one is impressed both by the amazing progress of scientific achievement and the very able manner in which that progress is portrayed.

F. E. Carr